Sofianopoulou, Stella and Mitsopoulos, Ioannis (2021) A review and classification of heuristic algorithms for the inventory routing problem. International Journal of Operational Research, 41 (2). pp. 282-298. ISSN 1745-7653

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A review and classification of heuristic algorithms for the inventory routing problem

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Abstract: The inventory routing problem (IRP) is an integration of vehicle routing and inventory management problems. In the recent years, it has increasingly drawn the attention of the researchers because of its potentially significant practical value. The IRP is classified as NP-hard problem since it subsumes the vehicle routing problem (VRP). This fact led to the development of many heuristic or metaheuristic approaches, although a small number of exact methods have been introduced recently. Heuristic methods offer the advantage of shorter time scales, i.e., greater computational efficiency, on the expense of course of the accuracy of the results. The immediate trigger for this study is our concern about results validation, which has been debatable in early papers, and only recently a systematic effort to create a set of optimally solved benchmark instances has been made. This article presents the heuristic methods for solving the basic variants of IRP found in the literature, stressing the computational results and the solution verification approach, rather than the methodology of the algorithms. The paper concludes with a discussion on the quality of the performance assessment of the proposed algorithms.

Keywords: heuristic algorithms; literature review; results validation; inventory routing problem; IRP.

Reference to this paper should be made as follows: Sofianopoulou, S. and Mitsopoulos, I. (xxxx) 'A review and classification of heuristic algorithms for the inventory routing problem', *Int. J. Operational Research*, Vol. X, No. Y, pp.xxx–xxx.

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1 Introduction

In general, inventory routing problems (IRPs) are defined on a graph $G = \{V, E\}$ where $V = \{0, \ldots. n\}$ is the set of vertices and $E = \{(i, j): i, j \in V, i \neq j\}$ is the set of edges. Each edge has a routing cost of c_{ij}. The set $V' = V\backslash\{0\}$ denotes the customers where vertex $\{0\}$ represents the supplier. Inventory holding costs are h_i, $i \in V$ per unit per period for the supplier as well as the customers. Each customer k has inventory capacity of C_k, $k \in V'$ whereas the supplier inventory capacity is C_0. The inventory levels at the end of period t of the supplier and costumer k are I_0^t and I_k^t, respectively. The length of the planning horizon is p, which means $t \in \{1, \ldots, p\}$. A set of M vehicles, $m \in \{1, \ldots, M\}$, with capacity q_m is available for the product distribution.

At the start of period 1 inventory levels of the supplier and the customers are known and equal to I_0^0 and I_k^0. In deterministic models, customer demand d_k^t is known beforehand for each customer k for every period t. However, in stochastic models, customer demand is only known in a probabilistic sense which is the main characteristic of stochastic inventory routing problem (SIRP).

The objective of the problem is to minimise total inventory and transportation cost while:

a the demand of each customer must be met

b the inventory capacity of each customer cannot be exceeded

c inventory level is not allowed to take negative values

d the capacity of each vehicle cannot be exceeded

d a single vehicle can perform only one route starting from the supplier, delivering to a subset of customers and returning back to the supplier.

The solution is comprised of three components:

- time of the delivery for each customer
- quantity of the delivery every time a customer is served
- delivery route for each vehicle.

Part of the IRP defined above is the classical vehicle routing problem (VRP). As a matter of fact, the IRP degenerates into a VRP if there is only one period in planning horizon, inventory costs are equal to zero, all customers need to be served during this period and vehicles capacities are infinite. Therefore, it is classified as an NP-hard problem and as a result it is usually solved with the use of heuristic algorithms.

In practice, several practical problems have benefitted from the applications of theoretical IRP models and their solutions. Thus, Accorsi et al. (2017) adopted a theoretical IRP model in a regional retailer supply chain where a VRP was tailored to optimise a food vendor's network. In their paper, Seyedhosseini and Ghoreyshi (2015) present a new model formulation for integrating production planning and distribution planning of perishable products through lot sizing and IRP. Also, in Montagné et al. (2018), an IRP is tackled over a time horizon using a constructive heuristic based on the shortest path and split procedures in which wasted vegetable oil has to be collected periodically from source points with varying characteristics. Alvarez et al. (2018) discuss an IRP variant with a single item, a single supplier, multiple vehicles and a finite multi-period planning horizon, minimising the sum of inventory and travel costs employing a metaheuristic algorithms based on iterated local search (ILS) and simulated annealing. Also in Matthopoulos and Sofianopoulou (2018), the authors discuss a variation of VRP, the heterogeneous fixed fleet VRP in which the vehicles available for distribution activities are characterised by different capacities and costs and the problem is tackled using a hybrid firefly algorithm.

The purpose of this paper is to review the literature on IRP variants close to the basic IRP, focusing on heuristic algorithms as a means of solving the problem. The emphasis will be on the computational results rather than on solving methodologies or deviations from the basic problem. The reviewed literature will be classified into four categories of increasing results credibility. These categories are:

- sensitivity analysis/assessing the impact of various parameters
- performance testing using various algorithms/models
- performance testing against lower bound (LB) and upper bound (UB) obtained from commercial software (MIP solvers)
- benchmark against optimal solutions.

Obviously, some research efforts may fall into more than one category. As a result, a research effort that uses various validation methods will be classified into the category of the most credible method employed.

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2 Classification of the papers according to validation method

2.1 Sensitivity analysis/assessing the impact of various parameters in random or real life instances

Anily and Federgruen (1990) implemented the first clustering heuristic to solve the IRP. The planning horizon is infinite and the unsatisfied demand is considered as lost sales. Each customer is considered to consist of several demand points of equal demand. Retailers are separated into clusters and part of their demand can belong to different regions, which also means it is served by different vehicles. Each time a delivery is made by a vehicle, it visits all of the customers in the specific cluster.

For a given partition of retailers, the remaining problem reduces to a separate constrained EOQ problem for each region. The authors derive a LB and an UB which are proven to be asymptotically optimal. The combined routing and replenishment strategies algorithm (CRRSA) starts by calculating the LB. Then, a subroutine partitions the retailer set and solves the travelling salesman problem (TSP) for each cluster. Last, a solution, which is also the UB, is calculated.

The CCSRA is tested in 136 randomly generated instances divided in nine categories designed to assess the impact of various parameters. The ratio (UB / LB) is used as an UB of the optimality gap since an exact solution could not be calculated at the time. The asymptotic optimality is verified in the results where even for small sized instances the ratio (UB / LB) is calculated low.

Bertazzi et al. (2002) consider the generic version of IRP described in the introduction with a single vehicle. An up-to-order policy is applied which is a limiting factor of the set of feasible solutions. A two-step heuristic is designed to solve the problem. In the first phase, an initial solution is constructed through the solution of an acyclic network for each retailer. The second phase is introduced to improve the initial solution by choosing every possible pair of retailers from the route to be removed and reassigned by the same procedures the initial solution was built. When a smaller total cost solution is determined the search stops. The authors also consider three variants of the original problem where the objective function includes individually the components: the sum of supplier inventory and transportation costs (variant 1), the transportation cost only (variant 2) and the retailer inventory cost only (variant 3).

The heuristic algorithm is programmed in FORTRAN to solve a series of randomly generated instances. Different parameters were combined into 24 cases. For each combination, a series of ten instances is generated.

First, the total cost distribution on its three components is calculated in three pairs of instances, where transportation, retailer and supplier inventory cost are taken into account. Each parameter is given a 'high' and a 'low' value. Second, a sensitivity analysis is conducted to determine the impact that parameters such as transportation cost, retailer and supplier inventory cost, minimum retailer inventory level and transportation capacity have on the total cost. Next, the heuristic algorithm is employed to solve the three variants of the original problem.

In the absence of exact optimal solutions and LBs, the authors compare the results from the heuristic algorithms with the total cost of two intuitive policies in eight randomly generated instances. Both policies require the exact solution of the corresponding TSP. The proposed heuristic performs better in both cases.

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Zachariadis et al. (2009) study a basic IRP variant with a finite planning horizon repeated to infinity and with no inventory held at the central depot. The special feature of this research effort is that the solution methodology is not limited to specific inventory policies examined in previous research works.

The proposed iterative heuristic starts by creating an initial solution with the help of a construction heuristic. Next, two local search (LS) operators are employed alternatively to either insert or remove a replenishment point from the current solution. A tabu search algorithm is then applied to improve the routing part of the solution. The procedure stops after the pre-specified time limit has been reached.

Ten instances were generated with varying number of customers, maximum demand, fleet size, vehicle capacity and customer dispersion around the depot in order to evaluate the performance of the algorithm. The planning horizon comprises of seven periods (one week).

The algorithm was programmed in Visual C#. The authors report the average elapsed time and total cost for each instance as well as the lowest cost with its corresponding elapsed time, demonstrating the reliability and robustness of the proposed methodology.

Geiger and Sevaux (2011) consider a bi-objective IRP where two objective functions representing inventory and transportation cost separately are considered. The aim is to identify Pareto-optimal solutions. Starting from a set of initial alternatives, the algorithm implements a LS strategy in order to discard dominated solutions. The method is exploring neighbouring solutions in two phases. First, a delivery schedule is planned and in the second phase, a capacitated VRP is solved by a savings heuristic or a record-to-record travel algorithm. A set of alternative solutions is continuously updated until it contains only non-dominated solutions. In order to reduce computational time, the concept of reference points is introduced where only a representative subset of the solution set is selected for improvement.

The authors use the geographical data of 14 instances found in Christofides et al. (1979) to generate a series of instances regarding the IRP. Planning horizon is set to 240 periods while three demand scenarios are proposed. The proposed solution approach is applied on four selected instances taking into account every demand scenario, i.e., a total of 12 problems are solved for three different numbers of reference points.

Results show that some scenarios are solved faster with the number of reference points having a substantial effect on computational times. A detailed analysis of the generated Pareto sets for four instances and for different number of reference points is then performed indicating that a first approximation with only a few reference points could provide useful results.

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2.2 Performance testing using various algorithms/models

Ribeiro and Lourenço (2003) discuss a partially SIRP where customers have either deterministic or stochastic demand, and inventory handling and stockout costs involved in the latter case. There is a fixed cost per vehicle used and the planning horizon is five days (week period).

The heuristic algorithm proposed reaches an initial solution to the inventory problem alone by minimising a complex cost expression using Gauss-Newton method. Next, a VRP is solved for each day applying an ILS method to minimise transportation cost.

An experiment is conducted where two different heuristic algorithms are used to solve the VRP, i.e., Clarke-Wright (CW) algorithm and ILS heuristic. For each instance, both algorithms are executed considering a different percentage of customers to exclude from the ordered cost list at each step of the algorithm. The problem instances used vary according to the type of the demand and the type of location.

At first, the authors compare the trade-off between transportation and inventory cost. The comparison suggests that there is an obvious tendency to increase the inventory cost while decreasing transportation cost. The performance evaluation of the two heuristics shows that ILS gives always better solutions at the expense of increased running time.

Campbell and Savelsbergh (2004) are focusing on distribution of industrial gases using tanker trucks. The goal is to minimise the distribution cost for the vendor only suggesting delivery quantities over a long-time horizon. A two-phase heuristic is proposed by the authors. In the first phase, an integer programming method is used to determine the customers to be served and the quantities to be delivered. The second phase specifies delivery times and customer sequences by using a greedy randomised adaptive search procedure (GRASP).

Large-scale real life instances are used as input data. A greedy algorithm is employed to test the computational results produced by the proposed IP-based algorithm. The greedy algorithm only contains a part of the considerations the authors make, so the parameters of IP-based algorithm are modified accordingly. The results indicate that the proposed method produces sufficiently better solutions. Next, emphasis is given at flexibility during routing and scheduling which give slightly better solutions. Finally, it is shown that GRASP method can improve the characteristic values as the number of iterations of routing and scheduling heuristic increases.

Aghezzaf et al. (2006) considered a long-term IRP to develop a cyclical distribution plan of a single product from a central depot to a set of retailers with the use of vehicle single/multi-tours without customer stockouts that minimises fleet operating and average total distribution and inventory holding costs.

An approximation algorithm based on column generation method is employed to solve the problem. A master problem is formulated which then is degraded and takes the form of an LP relaxation of the initial problem. After the LP problem is solved, a savings-based algorithm is used to produce multi-tours needed for the master problem to reach a solution in reasonable computational time.

Computational experiments conducted to test their method demonstrate the benefits from implementing the multi-tour approach instead of the single tour model.

Raa and Aghezzaf (2008) extended the previous work of Aghezzaf et al. (2006) to tackle a more plausible problem with constraints regarding customer inventory holding capacity, loading and unloading extra times and minimum times between consecutive deliveries. The concept of distribution pattern is introduced where a short route with high demand customers is executed more often than a long route involving customers of low demand. A delivery schedule is planned with the use of an EOQ-like model. During the solution process, a number of nested sub-problems are solved. The solution follows approximately the same steps as in Aghezzaf et al. (2006) where a heuristic based on column generation and a savings algorithm is utilised.

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A number of problem instances were generated to evaluate the performance of the proposed method. The problems include instances with varying number of customers, vehicle capacity and fixed cost, and last, optional customer inventory restriction. The commercial solver CPLEX 9.0 was employed for the column generation. Computational experiments show that the column generation-based heuristic solution approach finds the appropriate cost trade-off under varying circumstances. Furthermore, when fleet sizing is not considered, the heuristic outperforms an existing heuristic in finding the two way trade-off between distribution and inventory costs.

Savelsbergh and Song (2008) attempted to solve a variant of the IRP which differs in several ways from the standard problem. At first, there exist many depots instead of a central warehouse. Second, there might not be sufficient availability of the product at the depots and third, vehicle routes can expand in more than one time periods. This problem is defined as IRP with continuous moves.

The proposed solution approach is mainly concerned with developing an integer programming-based optimisation algorithm. Nevertheless, it is next combined with a LS scheme initialised by a randomised greedy heuristic (RGH) because of the excessive computational times required to reach a solution. The authors present an integer multi-commodity flow formulation on a time-expanded network in which activities of a vehicle are seen as a commodity and the network nodes represent a visit to a site at a particular time. The size of these networks in terms of variables become prohibitive and the use of customised IP techniques and search strategies are imperative in order to enhance solutions.

This IP-based optimisation approach has little to offer in solving real life instances despite its good performance in small problems. Thus, the authors suggest a combination with an RGH as in Savelsbergh and Song (2007), where an initial solution provided by RGH is improved using IP optimisation. Apparently, it is a case of a neighbourhood search concept where IP is used to explore different parts of the solution space.

Several computational experiments have been made to evaluate the performance of the IP algorithm. CPLEX 9.0 has been selected as an IP solver. Data from Praxair Inc. were used regarding production rates at plants, consumption rates at customers, vehicle capacities and customer inventory capacities. The base instance included seven production facilities, 200 customers and seven homogeneous vehicles. Three set of problems with varying size have been derived from this instance. Small problem instances were used to analyse the performance of the various settings of a branch-and-cut algorithm employed. Medium sized problems generated are used to compare the optimisation approach with the RGH. It is shown that IP algorithm gives almost always better results than the RGH for medium size instances. Instances of larger size were used to assess the possibility of embedding IP-based optimisation in a LS scheme in order to improve an initial solution found by RGH. In this latter case, the results indicate a small improvement upon the initial solution of RGH.

An IRP with an infinite planning horizon is considered by Zhao et al. (2008). The major difference from the basic IRP is that there is a three-level supply chain (supplier, warehouse and customers) instead of the classical two level chains. The goal is to minimise total long-run cost which includes ordering cost, holding cost (warehouse-retailers) as well as fixed and variable transportation costs.

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The authors apply a combination of the fixed partition policy proposed by Anily and Federgruen (1993), which divides the customer set into clusters and POT policy (Roundy, 1985) which under circumstances, calculates close to optimal replenishment intervals. Given a set of retailer partitions, a particular POT policy is calculated. In order to explore the solution space, a variable large neighbourhood search (VLNS) algorithm is used to find the optimal retailer partitions in which the POT policy is applied. In this VLNS scheme, the neighbourhood of the current solution is constructed by inserting random set of vertices into its neighbour routes, where the neighbour routes of each vertex are chosen randomly inside a given range. The number of randomly chosen vertices and neighbour routes may vary during the execution of the algorithm. Other techniques are also used during VLNS to boost its efficiency, such as searching level concept or a more sophisticated objective function.

The computational experiments are conducted in two phases. At the first phase, the computational experiment evaluates VLNS performance against a tabu search-based algorithm and a LB (Zhao et al., 2007). For this reason, VLNS algorithm is modified to solve the two echelon problem. Results show that VLNS algorithm gives slightly better solutions, most of the times, in a sufficiently better computational time.

At the second phase, after a LB is determined for the three echelon problem, a relation between the optimal solution and the LB is deduced. The conclusions drawn, characterising the relation between the optimal solution and the LB are consistent with the results, indicating thus the effectiveness of the method.

Boudia and Prins (2009) tackle an integrated production distribution problem without customer inventory cost or shortages. The central production facility has a limited daily production capacity and incurs a production setup cost as well as an inventory holding cost. The authors apply a memetic algorithm with population management (MA|PM), which is a rather recent metaheuristic to solve this special IRP variant.

Memetic algorithms are evolutionary population-based algorithms which use LS procedures to individually improve solutions. The metaheuristic employed is enhanced with the concept of population management (PM). In MA|PM, the mutation operator is replaced by a diversity control scheme based on a measure of distance between the solutions.

The computational results are benchmarked against results from previous work of the authors when a GRASP (Boudia et al., 2007) and a traditional two phase heuristic (H1), where vehicle routes are determined after the production planning (Boudia et al., 2005) were implemented. The tests conducted demonstrate that the proposed algorithm can tackle large instances in reasonable amounts of time.

A long-term IRP is studied by Raa and Aghezzaf (2009) an as in previous research efforts of the authors, a cyclic solution approach is adopted to solve the problem. Several realistic side constraints are added to their model. A column generation framework is used as the basis of the solution model which incorporates several heuristics such as insertion, savings and improvement algorithms. According to the authors, a distribution pattern is the scheme where a single vehicle executing repeatedly a set of different tours per period, with varying frequencies and at a certain cost rate. Heuristics are used to determine a set of distribution patterns. Next, the column generation process selects a subset of the produced distribution patterns that constitute a satisfying feasible solution.

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In order to assess the performance of the solution method proposed, a number of test instances are randomly generated. The random problems are created considering three factors. These are, customer storage capacity restriction (optional), inventory holding cost rate and the number of customers. The column generation part of the solution was undertaken by CPLEX 9.0. It is revealed that the two-way interaction of the three factors has a profound effect on the total cost. Furthermore, when fleet sizing is not considered, the heuristic is compared to the heuristic developed by Viswanathan and Mathur (1997). The results show that for every instance the column generation-based heuristic gives better solutions than the Viswanathan and Mathur (1997) algorithm.

Michel and Vanderbeck (2012) consider an IRP where deliveries to customers are replaced with pickups from various sites. Each site accumulates stock over time and it is emptied on each visit, corresponding thus to an order-up-to level (OU) policy. In addition, there is a dumping site at the end of each vehicle route. Inventory policy imposes that capacity overflows are not allowed, while pickups costs have to be minimised. Moreover, inventory management costs are not considered in contrast to stock transportation costs. Although the planning horizon is infinite, a periodic solution is sought.

The authors partition the customer set into clusters which are visited by only one vehicle per period and routing costs are approximately calculated. A column generation method is employed, combined with LS and rounding heuristics (RH). Specifically, a Dantzig-Wolfe decomposition algorithm is adopted. The problem decomposes into a master program, where inventory planning is tackled and into sub-problems, which determine vehicle routes.

The constructed algorithm is applied to a real life, large-scale industrial instance. First, the algorithm is executed with partial branching and cutting planes to produce the best possible heuristic result which requires excessive computing time. Then, the algorithm is implemented with different combinations of applied heuristics (LS, RH) and frequency of pickups (periodicities) and the solutions produced are compared to the result obtained from the first execution. Improved efficiency and computational times are reported.

Next, the solution with high pickup frequency is benchmarked against an industrial real life problem solution. The comparison is made in terms of the total distance travelled, the number of vehicles used and the average customer visits per week. It is shown that the proposed method produces better solutions with fewer vehicles utilised together with increased customer pickups. The latter is due to the fact that vehicles tend to visit customers before reaching maximum inventory capacity thus resulting in shorter travel distance.

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2.3 *Performance testing against LB and UB obtained from commercial software*

Abdelmaguid (2004) studied an IRP problem which considers inventory holding, transportation cost and backordering. The demand of each customer per period is small relatively to vehicle capacity and the distance between customers is not excessively large. The solution heuristic proposed assumes that product deliveries are performed only when customer's inventory reaches zero. This forbids partial fulfilment of a customer's demand which could lead to non-optimal solutions. A construction heuristic algorithm (approximate transportation cost heuristic – ATCH) has been proposed and implemented to tackle this problem. Two sub-problems, comparing inventory holding and backlogging decisions with allocated transportation cost estimates, are formulated and their solution methods are incorporated in this constructive heuristic.

The author produced two versions of ATCH. The first (ATCH-DP) solves optimally SUB2 using a dynamic programming algorithm. The second (ATCH-G) employs a greedy algorithm to solve SUB2. These heuristics are benchmarked against a simpler heuristic (multi-period vehicle routing problem – MPVRP) that prohibits inventory to be carried from each day. Moreover, these three heuristics are compared to LB and UB provided by CPLEX 8.1.

A number of randomly generated problems have been solved using these four methods. The percentage gaps between the total cost of each heuristic and the LB and UB obtained by CPLEX are used as performance indicators. Computational times are negligible, most probably because of the generally small size of the generated problems, with ATCH-DP performing the worst which is some seconds for larger problems. Of the three heuristics, ATCH-DP provided the lowest costs in most cases with MPVRP giving the worst results. The performance of ATCH-DP and ATCH-G are very close which implies that the greedy algorithm used to solve SUB2 is sufficiently efficient.

Furthermore, the deviation from the LBs for the two ATCH heuristics does not increase significantly along with the problem size. For larger problems, it is shown that ATCH-DP and ATCH-G outperform considerably CPLEX UBs. More specifically, for small problems, the LB percentage gaps lie below 20% for both heuristics and remain about this level for larger instances. Although ATCH-DP performs better than ATCH-G in small problems, the results tend to converge as the size of the problem increases. The results of MPVRP were steadily above 20% for any problem size but it outperformed CPLEX UB for large sized problems as was the case with ATCH heuristics.

Abdelmaguid and Dessouky (2006) attempted to outperform the construction heuristic algorithm proposed by Abdelmaguid (2004). In the author's earlier work, the possibility of partially fulfilling a customer's demand was not taken into account, which can result in poor solutions when customer order product quantities that are not significantly less than vehicle capacity. The authors introduced a genetic algorithm (GA) specifically designed to correct the flaws related to the previous approach. The objective function includes transportation and inventory holding and shortage costs. Backlogging is taken also into consideration as in the previous research effort.

The GA solution is represented by a two-dimensional matrix and the focus is on delivery quantities and times. Vehicle routing sub-problem can be solved by any efficient polynomial time heuristic such as Clarke and Wright algorithm. The GA initialises, i.e., creates a pool of random feasible solutions, by using a randomised version of ATCH developed by Abdelmaguid (2004). As a crossover operator, the parent solutions are exchanging lines which can result in non-feasible solutions. If a non-feasible solution appears, this is fixed by adjusting the delivery quantities. The mutation operator applied later is designed to exchange deliveries randomly and to investigate solutions including partial deliveries. A simple roulette wheel selector is responsible for selecting parent solutions and the elitism concept is also utilised.

The computational results from the proposed GA are benchmarked against the results from the two versions of ATCH (ATCH-DP and ATCH-G). Results are also compared to LBs calculated by CPLEX 8.1. Random problems are generated and parameters are determined so that backorders are carried in the optimal solution. Random tests follow

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the pattern of Abdelmaguid (2004). Again, the difference percentage between the total cost calculated by the three heuristic algorithms and the solutions from CPLEX are used as performance indicators.

Computational results from GA show that on average the solutions generated are of better quality, compared to both versions of ATCH heuristic. The GA proposed is capable of producing results very close to optimality in small problems whereas larger problems are within 20% of the LB. Accordingly, it is deduced that partial deliveries substantially influence the total cost, providing thus savings in transportation and shortage costs.

Abdelmaguid et al. (2009) study an IRP variant, where backlogging is the main feature. The demand is deterministic and small in relation to vehicle capacity and customers are located closely so as to make suitable a consolidated shipping strategy. The objective function includes fixed vehicle usage cost, variable transportation cost, inventory holding cost and shortage cost.

A constructive heuristic (estimated transportation cost heuristic – ETCH) is designed to solve this version of IRP where optimal delivery quantities are calculated by balancing the trade-off between transportation and inventory costs. The main characteristic of this constructive heuristic is that partial fulfilment of demand in the studied period is not allowed. ETCH is composed of two sub-problems, namely SUB1 and SUB2. Sub-problem SUB1 decides the delivery amounts for customers in day t and whether to have backorders or not. Sub-problem SUB2 decides whether to use possible remaining vehicle capacity in day t such that future demand is covered. The limitations of the constructive heuristic are overcome by an improvement heuristic which allows partial fulfilment and a more thorough search of the solution space.

The authors execute two versions of ETCH, namely ETCH-O and ETCH-H. These two versions are distinguished by the different solution methods for SUB1 and SUB2. Two scenarios are then considered. The first is used to assess the inventory holding decisions of ETCH whereas the second has its parameters appropriately modified so as to make backordering economically efficient and thus to evaluate backordering decisions of ETCH algorithm. It is reported that ETCH-O provides better solutions than ETCH-H. Nevertheless, computational time of ETCH-O is going to be significantly higher than ETCH-H as the problem size increases (number of customers, number of periods). The improvement heuristic increases the performance. Optimality gap of the ETCH heuristic increases together with the size of the problem at a constant rate.

Consistency in a multi-vehicle IRP is the subject of Coelho et al. (2012b). The term ‘consistency’ corresponds to the quality of service standards and refers to the quantities delivered, delivery frequency, vehicle load and fleet size. The only difference in problem formulation from Coelho et al. (2012a) is that multiple vehicles are used instead of a single vehicle resulting in a multi-inventory routing problem (MIRP).

The problem is solved with respect to the six consistency features involved. First, an adaptive large neighbourhood search (ALNS) algorithm is employed to determine vehicle routes through implementing a number of operators. Second, a mixed integer linear program (MILP) is solved by a minimum cost network flow algorithm to find the optimal delivery quantities. Next, a second MILP is solved to optimality by removing and reinserting operations in a given solution in order to improve results.

The instances reported in Archetti et al. (2012) have been used for the evaluation of the heuristic method proposed. It is clear that ALNS heuristic produces significantly

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better solutions. Results from the consistent form of MIRP are compared to the solutions of the basic MIRP in order to assess the impact of the various consistency parameters. It is found that imposing restrictions on the quantities delivered increases the cost and that applying the OU policy increases also average solution. Imposing consistency in vehicle capacity utilisation rate proves to be expensive, whereas consistency in driver assignment does not have major effect on the solution cost. Finally, a sensitivity analysis on the modifiable consistency features is reported. The analyses reveal that cost changes are of minor importance and follow the directions of the parameter variations.

2.4 Benchmark against optimal solutions

Archetti et al. (2012) introduced a heuristic in which mixed integer linear programming (MIP) is combined with a tabu search scheme. The present variant of the IRP is simple. There is only one vehicle starting from a central depot and customer stockouts are not allowed. The problem is solved considering two distinct replenishment policies: OU and maximum level (ML). The objective function is comprised of variable transportation cost and inventory holding cost of depot and customers.

The algorithm proposed is called hybrid heuristic for inventory routing (HAIR). First, the algorithm starts with an initialisation procedure to create a solution. Second, this initial solution undergoes a neighbourhood exploration and reaches the local optimum. A Lin-Kernighan optimisation algorithm is then applied to further improve transportation cost. Next, two mixed integer programs: MIP1 (route assignment) and MIP2 (route merging) are applied to this solution for further improvement. Last, the algorithm moves to a new solution using the current solution as a basis and then tabu search iterates. The proposed heuristic is benchmarked against instances with known optimal solutions. The heuristic algorithm is successful in finding optimal solutions for OU and ML policies. The algorithm is also tested to assess the impact of the improvement phase proposed which tries to improve a given feasible solution by solving a sequence of MIPs. The solution approach is also slightly modified and tested on larger instances for each replenishment policy considered.

Coelho et al. (2012c) consider an IRP which allows transhipment (IRPT) between supplier and customers (direct shipping) or between customers. There is a single vehicle in the model and inventory is not allowed to take negative values. The author considers two replenishment policies, namely OU and ML. The cost function includes distribution costs, transhipment costs and inventory holding costs at the supplier and at the customers as well. Transhipment costs are volume and distance dependent in contrast to distribution costs which are only distance dependent.

The problem is solved by using an ALNS metaheuristic. The operators of ALNS determine vehicle routes which transform the remaining problem to a minimum cost network flow problem. The resulting ALNS algorithm is modified appropriately to solve IRP with or without transhipment for both replenishment policies.

The minimum cost flow problem was solved using the scaling push-relabel algorithm. For the evaluation of ALNS heuristic, the researchers used instances with known optimal solutions presented in Archetti et al. (2007). It is shown that solutions provided by ALNS were cost effective for most instances. Comparative tests on a large set of artificial instances have shown that our heuristic can produce high quality solutions within reasonable computing times. Next, the impact of transhipment cost is assessed with regard to transportation cost. It is found that if transhipment cost is 10% of transportation

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cost the algorithm tends to disregard the transhipment option. The various operators of ALNS are also evaluated in terms of solution quality and running time.

3 Discussion on the classification of the IRP literature

The literature on heuristic methods for the basic IRP has been classified into the following seven categories:

a assess parameter impact in random instances

b assess parameter impact in real life instances

c assess parameter impact in instances solved to optimality

d sensitivity analysis

e performance testing using various algorithms/models

f performance testing against LB and UB obtained from commercial software

g benchmark against optimal solutions.

The first four of these have been collectively presented in Section 2.1 and deal mainly with checking the robustness of the proposed algorithms by examining the impact of various parameters or, in some cases, conducting a sensitivity analysis on the resulting outcome. This kind of assessment is common ground for most of the researchers and only few have been exclusively restricted to this way of evaluation (Anily and Federgruen, 1990; Bertazzi et al., 2002; Zachariadis et al., 2009; Geiger and Sevaux, 2011). Most of the research efforts have concentrated on comparing their results against different models, algorithms or algorithm versions as a more applicable way of validation (Ribeiro and Lourenço, 2003; Campbell and Savelsbergh, 2004; Aghezzaf et al., 2006; Raa and Aghezzaf, 2008, 2009; Savelsbergh and Song, 2008; Zhao et al., 2008; Boudia and Prins, 2009; Michel and Vanderbeck, 2012). On the other hand, the comparison against LB and UB calculated by commercial MIP solvers seems to be rather unattractive due to the required flexibility of any given heuristic algorithm. Therefore, the number of researchers that have taken this step is relatively small (Abdelmaguid, 2004; Abdelmaguid and Dessouky, 2006; Abdelmaguid et al., 2009; Coelho et al., 2012b). Finally, the cases that researchers have benchmarked their computational results against optimal solutions are quite a few (Archetti et al., 2012; Coelho et al., 2012a; Coelho and Laporte, 2013, 2014; Desaulniers et al., 2016). The reason for this is that optimal solutions have been available relatively recently and that there are inherent limitations to their use as benchmarks. In Table 1, papers on basic IRP are presented with respect to the best validating method together with the other methods used by the authors. It is evident that benchmarking computational results for the basic version of IRP in an undisputable manner is a challenging task and that the most accessible form of result certification is the comparison against previous models or algorithms.

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Table 1 Classification of the IRP literature using the most credible validating method

Increasing credibility →			
Sensitivity analysis/assess parameter impact	*Performance testing using various algorithms/models*	*Performance testing against lower and upper bounds obtained from commercial software*	*Benchmark against optimal solutions*
Anily and Federgruen (1990) [a]			
Bertazzi et al. (2002) [a, d]			
	Ribeiro and Lourenço (2003) [a, e]		
	Campbell and Savelsbergh (2004) [a, b, e]		
		Abdelmaguid (2004) [e, f]	
		Abdelmaguid and Dessouky (2006) [e, f]	
	Aghezzaf et al. (2006) [e]		
	Raa and Aghezzaf (2008) [a, e]		
	Savelsbergh and Song (2008) [a, b, e]		
	Zhao et al. (2008) [a, e]		
		Abdelmaguid et al. (2009) [a, e, f]	
	Boudia and Prins (2009) [e]		
	Raa and Aghezzaf (2009) [a, e]		
Zachariadis et al. (2009) [a]			
Geiger and Sevaux (2011) [a]			
			Archetti et al. (2012) [c, e, g]
			Coelho et al. (2012a) [c, e, f, g]
		Coelho et al. (2012b) [a, d, f]	
	Michel and Vanderbeck (2012) [e]		

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4 Conclusions

In this paper, the existing literature regarding the efficiency of heuristic algorithms in solving the IRP is reviewed. The focus was on research efforts solving the basic version of IRP, with minor deviations. Our survey referred not only to the specific results produced by each heuristic algorithm but to the certification of the proposed solutions as well.

Until recently, most researchers could not use a set of benchmark IRP instances, solved to optimality, to test their results. This fact led them to use LB and UB, provided by IP commercial solvers like CPLEX, to determine an optimality gap. However, because of the limitations of the IP solvers, some authors resorted to the comparison between their algorithms and previous work. There were only a few real life industrial problems that were used as benchmarks to test the effectiveness of proposed solutions, which can be explained by the complexity of real life problems and the reluctance of the companies to share information. In absence of actual benchmarks, testing the algorithm in several scenarios gave the opportunity to make a rough estimation of the heuristic robustness and behaviour.

A number of exact methods have been introduced lately (Archetti et al., 2007; Solyalı and Süral, 2011; Adulyasak et al., 2012; Coelho and Laporte, 2013) whose results can be used as benchmark points. Nevertheless, there are limitations in their use as benchmarks because of the manifold nature of the IRP. For example, the exact methods that have been developed consider only one vehicle (Archetti et al., 2007), implement only an OU policy (Solyalı and Süral, 2011) or do not consider backlogging (Coelho and Laporte, 2013). Therefore, a heuristic algorithm proposed has to be able to adapt to these benchmark instances before its actual implementation.

Finally, it should be noted that, in discussing the performance testing of proposed algorithms in the literature of IRP, we hope that the need for a proper treatment of quality of algorithmic performance assessment will be motivated.

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