The Use of Markov Chain Analysis for Rule-Based Power and Energy Management Optimisation in Electric Vehicles

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Abstract
In this paper the development of a Rule-Based Power and Energy Management Strategy as a result of Markov Chain analysis will be shown. Using real-world drive cycle data a Markov Chain Transition matrix is build from which a Bias matrix is developed showing the difference between acceleration and deceleration with respect to the next velocity as an extension to the Markov Chain. From this the parameters for a PEMS are developed, simulated and the results discussed and compared to other strategies.

Keywords: Energy, Optimisation, Power Management, DC-DC, Markov Chain, EV

1 Introduction
The battery pack in an EV is designed according to a power to energy ratio and is a trade-off in the design of the pack. The battery also suffers from effects such as rate capacity effect, ripple effects and inefficiency under charging. These effects result in losses through which the capacity and life span of the batteries are compromised, affecting range and drivability of the vehicle. Using a combination of Ultra Capacitors (UC) and a DC-DC converter (UC Module) it is possible to reduce the peak power as seen by the battery [1].

The aim of the Power and Energy Management Strategy (PEMS) is to optimise the power split between the battery and the UC Module and to increase energy efficiency and lifespan of the energy sources. This is considered to be of extreme importance for the acceptance of EVs [2-4]. In addition to the technical and practical reasons there are economical advantages to optimising the drive train such as reducing the energy components resulting in a cost reduction of the total drive train.

In this paper a new method to set the RB parameters for the Power Management Strategy (PMS) will be demonstrated, while the developed Energy Management Strategy (EMS) will provide smooth transitions and reduce susceptibility to noise. This method is based around the predictive capabilities of Markov Chain Analysis and more specifically the information it holds regarding the moments of acceleration and deceleration. The resulting Power and Energy Management Strategy (PEMS) is fast, easy to implement and provides a good result over different drive cycles.

2 Method
Using real-world drive cycle data, a Markov Chain Transition matrix was developed, which was used to calculate the probabilities at a number of different intervals. A Bias matrix was developed which shows the difference between acceleration and deceleration towards the next state velocity and provides a method of analysis of the acceleration and deceleration patterns. The Bias vector indicates whether the remainder of probabilities is biased toward acceleration or towards deceleration. The result of the Bias analysis then leads to the development of an EMS,
which can feed a PMS. A comparison is made between this rule based strategy and other energy management strategies. The final PEMS is simulated and the results are discussed. Section 3 describes the literature review. Section 4 describes the Markov Chain matrix development. Section 5 describes the EMS design, and section 6 describes simulation and the results. The conclusions are discussed in section 7.

3 Literature review
For hybrid electric vehicles control can be broken down to three levels [5, 6].

3.1 Operational (Power electronics)
The Operational level addresses the way in which the converters are switched. Accepted forms of control are voltage mode control and current mode control with or without a voltage control loop [7]. The choice for a particular scheme depends on the desired speed and stability of operation under changing loads.

3.2 Tactical (Power Management)
Tactical is the power split between different sources, it defines which source supplies a given amount of power at any given time and sets limits on how much power can be supplied by each source. The power management strategy is described in (1) where n is the number of available sources and \( P_k(t) \) is the power contribution of each individual source at a specific moment in time, which results in the total requested power \( P_T(t) \).

\[
P_T(t) = P_1(t) + P_2(t) + \cdots + P_n(t) = \sum_{k=1}^{n} P_k(t)
\]

A key aspect in the tactical control is the decision regarding when and how much to recharge the UC. In literature the reported value of kinetic energy that can be recovered is between 30-50% [8]. The battery will provide the missing charge. Charging an UC requires time and can be optimised if the required acceleration is known in advance [9].

Most driving patterns are, to a certain extent, unique (either because of route, traffic or other factors), therefore the UC module needs to be ready to be able to supply a certain value at any given time with the probability that at lower speeds an acceleration event is more likely than a regenerative event, while at higher speeds it is more likely for a regenerative event to occur. In addition, it is unlikely that an acceleration event always starts at 0 and ends at 70 MPH. The conclusion is that the converter does not need to be designed to supply up to the maximum acceleration, while the UC does not need to be designed to have the energy to cover a full acceleration profile.

3.3 Strategic (Energy management)
Strategic is the overall management strategy to ensure that the limitations of each source are not violated. The strategic control monitors the available energy in the different sources over time and decides on the best available strategy for the distribution of energy, which in turn informs the tactical management level on available power split options.

An optimisation strategy involves the optimisation of a cost function such as optimised fuel consumption, reduced peak demands, reducing weight without sacrificing other features such as driveability (acceleration and deceleration response) and safety. The number of variables can be numerous which will increase complexity of the equations. The best optimisation strategies are calculated offline and often rely on a priori knowledge of the proposed drive aspects and require large databases containing a number of look up tables which require seconds or even minutes to step through (stochastic optimal control, dynamic programming (DP)) [10, 11].

Model Predictive Control (MPC) aims to overcome the problems of DP by tuning the system offline and applying it online [12]. The tuning requires in-depth knowledge to undertake a number of difficult adjustments. Predictive control shows that while the possibility for optimisation is significant it requires large amounts of storage space or computational energy. Instead of storing the complete prediction profile supporting vector points are stored, which reduces the required storage space. In addition, a baseline strategy is necessary if the route had not been driven before [13].

Drive Cycle prediction based upon past events, while seemingly very suitable for recurring routes, fail to deal with traffic effects and weather influences [13]. Drive cycle knowledge is considered important for future optimisation strategies while manual tuning should be avoided [14].
Energy management strategies based on predictive control are complicated and often difficult to understand and modify. Also, they can be slow due to number of computations that are needed [15]. It is therefore important that energy management strategies also focus on the sub-optimal strategies.

Equivalent Consumption Minimisation Strategy (ECMS) aim to simplify the control problem by applying optimal combination of variables and is rated in optimisation capability close to MPC [11]. However, it does not examine driving behaviour or predict influences from outside [16]. Heuristic control strategies – Rule Based (RB) strategies do not actively search for the most optimised solution but assume a solution based on the limitations set [17]. The limitations are fixed points in the operation which results in susceptibility to noise. The addition of fuzzy logic allows for smoother transitions between operation points which improves continuity and robustness but at the expense of increase computation requirements and data storage [18]. RB control, such as Solid State Machines and fuzzy logic controllers have the advantage of being able to function in real-time and are robust but are not as rigorous in optimisation as for example a DP or MPC strategy [19]. Learning strategies such as Neural Networks (NN) promise good optimisation but are dependent on available training data [18, 20]. According to Gurkaynak, et al. [18] NN are better than RB strategies and can be further improved through fuzzy logic. A NN is not considered as good as a MPC [12].

Energy Management in vehicles can also be done through flexible electric load demand [21, 22] where the converter to charge the auxiliary battery is switched on and off as part of the load control. The auxiliary battery can sustain the load (from auxiliary equipment, such as radio, heaters, window wipers, light, electric windows, etc.) on its own for a limited amount of time.

In Table 3 a comparison summary of the different strategies is shown. The effectiveness of the optimisation is given a rating on the following scale: H = High, M = Medium, L= low. The required aspects are marked by an x. The -- marker is used to indicate an either or situation.

### 4 Markov Matrix Development

A study conducted by Knowles, et al. [23] presented a driveability study (11 participants) from which data was collected using a Mitsubishi iMiev electric vehicle, driven along a standardised route. The data was further analysed and converted in to a Markov Chain matrix.

If $M$ is the transition matrix of possibilities (2) then each value of the matrix is the probability $(p)$ of achieving the next state $(j)$ from the current state $(i)$ (3) [24].

$$M = \left[ p_{ij} \right]_{i,j=1,...,n} \tag{2}$$

Where:

$$p_{ij} = P[S(t+1) = S_j | S(t) = S_i] \tag{3}$$

An example matrix is given in (4) where the current and next state are given in km/h.

<table>
<thead>
<tr>
<th>Probability ($p_{ij}$)</th>
<th>Next State (km/h)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>5</td>
</tr>
<tr>
<td></td>
<td>10</td>
</tr>
<tr>
<td></td>
<td>125</td>
</tr>
<tr>
<td>Current State (km/h)</td>
<td>i_{26j1}</td>
</tr>
<tr>
<td></td>
<td>i_{26j2}</td>
</tr>
<tr>
<td></td>
<td>\vdots</td>
</tr>
<tr>
<td></td>
<td>i_{26j26}</td>
</tr>
</tbody>
</table>

The resulting matrix graph from the drive cycle data is shown in Figure 1. The data was rounded to the nearest multiple of 5 km/h because of the low number of participants.

**Figure 1:** Transition Matrix plot

Through the use of the transition matrix the probabilities were calculated at 5 second intervals as shown in Figure 2. It can be seen that over time there is a development of maximum value lines appearing through the matrix, which indicates, for example, that starting at a current speed between 30 and 60 km/h the next velocity will be
approximately 45 km/h. This trend is already visible at 20 seconds Figure 3.
While it is clear from Figure 2 that the highest probability results in a target speed value the probability of achieving that value is dropping quickly – from 50% at 5 seconds to 25% at 20 seconds (Figure 3) which means that at 20 seconds there a 75% chance that this value is not achieved.

The effect of this remaining percentage results in a bias towards either a probability of velocity reducing or increasing. Any current state resulting in a lower next state is indicative of a braking probability while when the next state is higher this is indicative of an acceleration probability. This BIAS probability is defined in equation (5). Where, $B$ is the Bias vector of the probability matrix at $x$ seconds; it is a measurement of difference. The Bias vector indicates whether the remainder of probabilities is biased toward braking or towards acceleration. A positive result indicates an acceleration event while a negative result indicates braking. The bias is calculated and compared to the maximum probability for the intervals 5 and 20 seconds and is shown in Figure 4 & 5. The values on the maximum probability line show the next state from the current state (the x-axis).

$$B = \begin{bmatrix}
\sum_{j=b_1+1}^{b_2} p_{1j} - \sum_{j=1}^{b_1} p_{1j} \\
\sum_{j=b_2+1}^{b_3} p_{2j} - \sum_{j=1}^{b_2} p_{2j} \\
\vdots \\
\sum_{j=b_{n-1}+1}^{b_n} p_{nj} - \sum_{j=1}^{b_{n-1}} p_{nj}
\end{bmatrix}$$

(5)

Where $[p_{ij}] = M^{(x)}$. 

Figure 2: Probability matrices at 5 seconds

Figure 3: Probability matrices at 20 seconds

Figure 4: Bias and Maximum probability at 5s

Figure 5: Bias and Maximum probability at 20s
While any maximum probability value after 20 seconds averages 20.86% the average probability value of any acceleration or braking events are 36.37% ($P_{\text{accel}} = 42.50, P_{\text{brake}} = 31.11\%$).

This information will allow us to slowly increase the battery supply over the set period of time while the peak demand is dealt with by the UC modules. The smoothing of the battery power demand is achieved according to:

- The velocity equates to a power demand (the assumption is made that there is no gradient)
- The BIAS matrix has resulted in a set of rules to which the UC target State of Charge (SoC) can be set.
- A filter function is based on the interval duration chosen (in this case 20 seconds)

5 Energy Management Strategy

As discussed earlier, the levels of control for any multiple energy systems are: 1) Operational, 2) Tactical, and 3) Strategic. The Operational level is dealt with through the chosen converter control, which in this case is peak current mode control with a slower voltage control loop.

The Tactical level is described by the power split between the battery and the UC Module. In [1] the authors describe and show through simulation a tactical control strategy which requires a value for battery contribution ($I_{\text{bat-max}}$) and a value for the UC target State of Charge ($UC_{\text{SOC}}$). This paper will use a similar setup. The EMS defines these two values as part of the Strategic operational level.

A 20 second time interval was chosen based on the behaviour of the Bias curve which had stabilised after this time period (no major changes). It was also felt that a longer period would be unrealistic for the UC Module to support because of the size of the module required. 20 seconds is also a critical duration when recovering energy. Any recovery taking place using a battery has an efficiency of less than 70% effective compared with recovery into a UC which is 95% efficient [25]. Based on the chosen (20 second) interval a filter is designed to simulate the slow rise and to serve as a reference for a second by second update of the battery power limit.

A second order Butterworth low-pass filter is chosen of which the generic form is given in equation (6). The Butterworth filter was chosen because of its flat response up to the cut-off frequency.

$$Y(s) = \frac{\omega^2}{s^2 + 2 \cdot \beta \cdot \omega + \omega^2} \quad (6)$$

Where $\omega$ is the cut-off frequency in radians per second and $\beta$ is the damping factor. The damping factor is set to 1 which results in a critically damped response thus providing a flat response.

The timing interval is 20 seconds but this is only one quarter of the total frequency. The total period to calculate $\omega$ (7) is therefore $T = 80$ seconds.

$$\omega = 2\pi f = \frac{2\pi}{T} \quad (7)$$

The UC maximum power contribution was calculated at 30kW and with a period of 20 seconds this results in 600kWs of energy. For the parameters of the UC, a Lithium-ion UC [26] was used. The UC pack was designed from 23 cells in series which provides a voltage range of 50.60 V – 87.40V and three strings in parallel which creates a capacity of 286.95F. The pack has an internal resistance of 10.73 mΩ.

Based on a small vehicle (1200 kg) the power demand under cruising was established as a maximum value for the battery power supply (Table 1) at intervals corresponding to the maximum value lines in the Markov Matrix. The maximum value of recoverable energy, available for the UC, has been calculated based on its kinetic energy value as per equation (8) assuming a 50% maximum energy recovery rate [8].

$$E_{\text{rec}} = \frac{1}{2} M v^2 \cdot 0.5 \quad (8)$$

Where $E_{\text{rec}}$ is the recoverable energy, $M$ = mass of the vehicle and $v$ is the vehicle velocity in meters per second ($\text{m/s}$). This information is then used to calculate the target $UC_{\text{SOC}}$ (Table 1).

<table>
<thead>
<tr>
<th>km/h</th>
<th>$I_{\text{bat-max}}$</th>
<th>$UC_{\text{SOC}}$</th>
</tr>
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<tbody>
<tr>
<td>0.00</td>
<td>0</td>
<td>87.40</td>
</tr>
<tr>
<td>20.00</td>
<td>1744</td>
<td>87.03</td>
</tr>
<tr>
<td>50.00</td>
<td>4617</td>
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<td>70.00</td>
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<td>17064</td>
<td>77.62</td>
</tr>
<tr>
<td>110.00</td>
<td>21313</td>
<td>75.41</td>
</tr>
<tr>
<td>120.00</td>
<td>26309</td>
<td>72.91</td>
</tr>
</tbody>
</table>
The Markov Chain transition matrix after a 20 second interval is shown in Figure 2b and the Bias is calculated and compared to the maximum probability at 20 seconds (shown in Figure 4b). Support from the UC Module is expected during this time interval. The values on the maximum probability line show the next state from the current state (the x-axis). While any maximum probability value after 20 seconds averages 20.86% the average probability value of any acceleration or braking events are 36.37% ($P_{\text{accel}}= 42.50, P_{\text{brake}} = 31.11\%$).

It is this improvement in prediction that allows setting of the $UC_{\text{SOC}}$ value based on the current velocity and the expected velocity that would result in a braking event and thus recovery of energy. This information further allows the slow increase of the battery supply over the set period of time while the peak demand is dealt with by the UC modules.

The smoothing of the battery power demand is achieved through the PMS which sets the allowed power demand from the battery and the $UC_{\text{SOC}}$. The designed filter function for the EMS controls the updating of the battery allowed maximum as well as the UC SoC target value based on the designed rules.

### 6 Simulation

In Matlab / Simulink a simulation was setup using two different electric vehicle drive trains. The chosen topologies are shown in Figure 6 & 7.

Simulations were conducted over four different drive cycles: two drive cycles derived from the research data on which this technique is based and two unrelated drive cycles: New York City Cycle (NYCC) and the New European Drive Cycle (NEDC) to see how this technique compares if the route were to change. The ECOp and ECOn drive cycles used in the simulations are the most ECO positive and ECO negative drive data sets from our standardised route. With the positive driver driving in a controlled manner and anticipating traffic situations while the negative driver was both accelerating and braking hard.

The improvement is measured as an average percentage improvement per second, equation (9).

\[
\text{Improvement per second} = \left( \frac{I_{\text{bat}1} - I_{\text{bat}2}}{I_{\text{bat}1}} \right) \times 100%
\]

Where $I_{\text{bat}1}$ and $I_{\text{bat}2}$ are the battery currents of the baseline topology and the comparing topology respectively at k intervals. Where k is a 1 second
interval rate and \( n \) is the final point of the drive cycle duration.

\[
\% = \sum_{k=1}^{n} \frac{I_{\text{bat1}}(k) - I_{\text{bat2}}(k)}{I_{\text{bat1}}(k)} \times 100\% \quad (9)
\]

Figure 9 show graphs of the filtering effect on the battery current for the different drive cycles. The average percentage improvements results are given in Table 2. The first two drive cycles ECO\text{on} and ECO\text{p} are based on the original data show almost a 50% improvement. The transferability of this PEMS to another drive cycle is shown through the outcomes from NYCC and NEDC. A large part of the NEDC is cruising at high speeds where reductions as a result of peak power smoothing are minimal. The PEMS would be considered a fast response.

The next step in development is to provide online adjustment of the rules which would make the system self learning at the expense of introducing a database. This would further optimise the use of available UC Module energy. This research shows that using the Markov Chain on a small data sample provides optimised results across a wide range of driving conditions. This is important because this means a potential database would not require a lot of space which would allow for online implementation. But further research in this aspect is required.

<table>
<thead>
<tr>
<th></th>
<th>% improvement</th>
</tr>
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<tbody>
<tr>
<td>ECO\text{on}</td>
<td>47.37%</td>
</tr>
<tr>
<td>ECO\text{p}</td>
<td>46.99%</td>
</tr>
<tr>
<td>NEDC</td>
<td>23.51%</td>
</tr>
<tr>
<td>NYCC</td>
<td>86.93%</td>
</tr>
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</table>

### 7 Conclusions

In this paper a method to use Markov Chain Analysis to establish the parameters of a RB PEMS is shown. The development method resulted in a PEMS with a flexible strategy and high...
percentage improvement. The developed RB PEMS provides smoothing against susceptibility to noise in a simple to implement method. The online use data sampling and application of Markov Chain would still require a database but the research shown here shows that its size can be limited. And it is not computationally intensive. The final row in Table 3 shows the rating of current proposed PEMS.

References


Authors

Dr. Theodorus (Dirk) Kok holds a PhD from the University of Sunderland and is currently researching low carbon vehicle topics such as the use of hydrogen in ICEs and improvements to the range and driveability of battery electric vehicles via power and energy management through the use of Markov Chain Analysis and Fuel Cell Auxiliary Power Units.

Adrian Morris has an MSc from the University of Sunderland and is an active researcher in the low carbon vehicle arena. His previous projects include the development of a dual fuel ICE powered vehicle and the conversion of battery powered electric vehicles to use fuel cells. He is the project lead at the University of Sunderland for the EU Interreg IV project Hyacinth.
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