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Multi-relation-pattern knowledge graph embeddings for link prediction in hyperbolic space

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ABSTRACT

The aim of Knowledge Graph Embedding (KGE) is to acquire low-dimensional representations of entities and relationships for the purpose of predicting new valid triples, thereby enhancing the functionality of intelligent networks that rely on accurate data representation. In recommendation systems, for example, the model can enhance personalized suggestions by better understanding user-item relationships, especially when the relationships are hierarchical, such as in the case of user preferences across different product categories. Existing KGE models mostly learn embeddings in Euclidean space, which perform well in high-dimensional settings. However, in low-dimensional scenarios, these models struggle to accurately capture the hierarchical information of relationships in knowledge graphs (KG), a limitation that can adversely affect the performance of intelligent network systems where structured knowledge is critical for decision making and operational efficiency. Recently, the MuRP model was proposed, introducing the use of hyperbolic space for KG embedding. Using the properties of hyperbolic space, where the space near the center is small and the space away from the center is large, the MuRP model achieves effective KG embedding even in low-dimensional training conditions, making it particularly suitable for dynamic environments typical of intelligent networks. Therefore, this paper proposes a method that utilizes the characteristics of hyperbolic geometry to create an embedding model in hyperbolic space, combining translation and multi-dimensional rotation geometric transformations. This model accurately represents various relationship patterns in knowledge graphs, including symmetry, asymmetry, inversion, composition, hierarchy, and multiplicity, which are essential for enabling robust interactions in intelligent network frameworks. Experimental results demonstrate that the proposed model generally outperforms Euclidean space embedding models under low-dimensional training conditions and performs comparably to other hyperbolic KGE models. In experiments using the WN18RR dataset, the Hits@10 metric improved by 0.3% compared to the baseline model, and in experiments using the FB15k-237 dataset, the Hits@3 metric improved by 0.1% compared to the baseline model, validating the reliability of the proposed model and its potential contribution to advancing intelligent network applications.

1. Introduction

A Knowledge Graph (KG) serves as a structured graph format for organizing knowledge facts, typically expressed as triples (head entity, relationship, tail entity), represented as (h, r, t). In this structure, entities are depicted as nodes, while the relationships are illustrated as edges. This representation form of abstract knowledge in the real world using graph data structures integrates heterogeneous data to some extent, addressing the challenges faced by artificial intelligence models in handling complex structured heterogeneous data. It has significant applications in information retrieval, intelligent question answering, recommendation systems [1–3], natural language processing, and other fields. In the context of intelligent networks, KG facilitates improved communication and interaction among interconnected systems, providing essential contextual information that enhances decision-making processes and optimizes network operations.

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However, both manually annotated data and knowledge graphs constructed using knowledge extraction models generally only represent a subset of real-world knowledge (i.e., they suffer from a large number of missing knowledge triples) and are often incomplete. This incompleteness is detrimental to various downstream tasks. Additionally, due to the vast amount of information available in the era of big data, manually completing knowledge graphs is impractical. Therefore, KG completion, which aims to predict new triples, has become an important research direction in recent times, with implications for enhancing the efficiency and robustness of intelligent networks.

In recent years, researchers have developed various knowledge graph embedding (KGE) techniques [4–6] to address the KG completion problem. One of the foundational models, TransE, mapped entities and relationships into a continuous Euclidean space, offering a simple yet effective solution for modeling basic relational patterns. Building on this, models like TransH extended the approach to handle multi-faceted relations by allowing entities to have distinct representations across different relationships. Further advancements such as RotatE introduced geometric transformations, representing relationships as rotations in complex vector space to effectively model symmetry and inversion patterns. These methods demonstrated the potential of Euclidean spaces for KGE but encountered challenges when representing hierarchical or complex graph structures.

To address these limitations, hyperbolic geometry has emerged as a powerful alternative. MuRP pioneered the application of hyperbolic embeddings, leveraging the properties of hyperbolic space to capture hierarchical relationships efficiently, especially in low-dimensional settings. Building on MuRP, HyperKG incorporated additional mechanisms to better model complex interactions in knowledge graphs, such as multi-relational projections. These advancements underline the continuous evolution of KGE techniques, with each iteration refining the ability to handle the inherent complexity and diversity of knowledge graph structures. Using the unique properties of hyperbolic space, modern KGE methods provide a robust framework for addressing challenges in knowledge graph completion and extending their applicability to intelligent networks.

The relationships in a knowledge graph mainly involve several relationship patterns, including symmetry, asymmetry, inversion, composition, hierarchy, and multiplicity, as illustrated in Fig. 1. The current popular embedding methods are mostly designed to model one or more of these relationship patterns. TransE represents relationships as translation operations on vectors, enabling the modeling of relationship patterns such as asymmetry, inversion, and composition. RotatE represents relationships as rotation operations on vectors, simulating relationship patterns such as symmetry, asymmetry, inversion, and composition. MuRP embeds relationships into hyperbolic space, allowing the model to better capture the hierarchical structure of relationships. Building upon these previous works, our proposed method, MrpHKGE, aims to achieve effective embedding by modeling all of the aforementioned relationship patterns.

In the domain of KGE, recent advancements have focused on more complex relationship patterns, including multi-relational embeddings and hierarchical data representations. Some models have extended Euclidean and hyperbolic spaces to handle these patterns more effectively. For example, tree-based embedding techniques have been employed to model hierarchical structures more explicitly. However, these models still face scalability and flexibility issues when it comes to representing complex multi-relational data. MrpHKGE addresses these challenges by learning separate curvatures for each relationship in hyperbolic space, offering a more scalable and flexible solution for handling complex hierarchical and multi-relation patterns in large-scale knowledge graphs.

Here, an explanation is given of how the proposed method (MrpHKGE) models all relationship patterns. Firstly, due to the excellent performance of rotation-based models in the embedding domain, this model employs both 2D rotation and 3D rotation methods to model

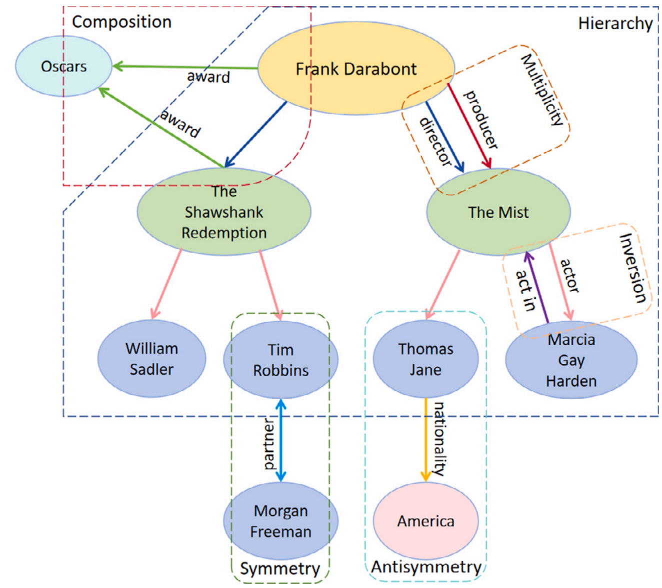

Fig. 1. Example Diagram of Relationship Patterns in Knowledge Graphs.

relationship patterns such as symmetry, asymmetry, inversion, and composition. The model combines an attention mechanism to evaluate the performance of different rotation methods on different data and achieve the optimal combination. Secondly, although individual translation-based models may not perform satisfactorily, they are indispensable for modeling multiplicity. This model combines the residual idea and implements skip connections by applying initial relationship vectors for translation before and after other geometric transformations. This approach not only improves the accuracy of the model, but also helps to prevent the disappearance of the gradient to some extent. Lastly, the model completes all the aforementioned geometric transformations in hyperbolic space. Trains separate hyperbolic space curvatures for each independent relationship in the knowledge graph. This allows the model to effectively capture the hierarchical information of the data, allowing clear differentiation between entities at different levels.

This study evaluates the proposed model, MrpHKGE, using experiments conducted on two widely recognized public datasets: WN18RR [7] and FB15K-237 [8]. The findings indicate that MrpHKGE exhibits performance on par with leading KGE models in low-dimensional embeddings. In some cases, it even surpasses mainstream models in terms of certain data and metrics. Additionally, the model exhibits similar performance in high-dimensional space, which confirms the effectiveness of the MrpHKGE model, thereby contributing to the advancement of intelligent networks that require dynamic and scalable data representations.

2. Related work

2.1. Euclidean embeddings

Many KGE models are designed within Euclidean space, primarily relying on operations such as translation and rotation.

Translation-based model. TransE [9]combines geometric transformations in space with embeddings, pointing out a new direction for the field of KGE. This model has led to the development of numerous translation-based models, such as TransH [10], TransR [11], and TransD [12]. The fundamental concept of translation-based embedding models is to interpret the relation r as a translation operation that connects the head entity to the tail entity, implying that the model transforms the head entity into the tail entity via the specified translation. TransH identifies the shortcomings of the TransE model in handling 1-N, N-1, and N-N relations and addresses these issues by introducing a specific relation hyperplane. TransR enhances TransH by expanding the hyperplane into a space that is specific to each relation. In turn, TransD takes a further step by employing two distinct vectors to represent both entities and relations within the knowledge graph.

Rotation-based model. The RotatE model [13] employs a more complex geometric transformation: rotation, for embedding. The QuatE model [14] utilizes the concept of quaternions to upgrade the 2D rotation transformation of RotatE to three dimensions, further improving the model's accuracy. The DualE model [15] employs dual quaternions to represent relations through a fusion of translation and rotation. These rotation-based embeddings have achieved better results than translation-based models, but such embeddings often require high-dimensional spaces, leading to significant memory consumption.

2.2. Graph neural network

ConvE [16] was the first to employ a convolutional architecture with a single convolutional layer to accomplish the knowledge graph completion task. It reconstructs the embeddings of the head and tail entities into 2D formats and uses them as inputs to the convolutional layer to obtain query embeddings. The output of the convolutional layer is vectorized through a linear transformation and matched with the tail entity, scoring the given triples and significantly accelerating training and evaluation speeds. ConvKB [17] is another KGE model that uses convolutional neural networks. It simplifies ConvE while retaining its translational properties through one-dimensional convolutions. CapsE [18] builds on ConvKB by adding a capsule neural network layer above the convolutional layer. Various graph-based convolutional neural networks, including R-GCN [19], SACN [20], and KBGAT [21], utilize graph convolutional networks, weighted graph convolutional networks, and graph attention networks, respectively, to derive embeddings for knowledge graphs.

2.3. Hyperbolic embeddings

In recent years, using hyperbolic embeddings to represent hierarchical data has shown great advantages in low-dimensional spaces, leading more researchers to apply hyperbolic embeddings to KGE. MuRP [22] is the first KGE model to utilize hyperbolic geometry, implementing the translational idea of the TransE model in hyperbolic space and optimizing the model using the concept of Riemannian manifolds. AttH [23] employs geometric transformations such as rotation and reflection for hyperbolic embeddings and introduces an attention mechanism to optimize the embedding model. Building on the AttH model, Huiru X et al. [24] introduced fast Fourier transforms to transform entity embeddings between different geometric spaces, achieving excellent results.

3. Preliminary knowledge

Before presenting the method proposed in this paper, it is necessary to explain the problem of KGE and the related methods in hyperbolic geometry.

3.1. KGE for link prediction

A knowledge graph is defined as a set of triples $(h, r, t) \in \varepsilon \subseteq \mathcal{V} \times \mathcal{R} \times \mathcal{V}$, where $\mathcal{V}$ represents the set of entities, and $\mathcal{R}$ represents the set of relations. KGE essentially involves mapping entities $(h, t) \in \mathcal{V}$ to embeddings h', t', and mapping $r \in \mathcal{R}$ to r', while maintaining the maximum amount of semantic and structural information from the knowledge graph.

Link prediction involves inferring the most likely unknown triples based on known information and ranking all candidate triples using a scoring function. Specifically, for a given triple (h, r, t), assuming any one element is missing, the embedding model will iterate through each entity or relation to fill in the missing part and calculate the score for the completed triple. The score indicates the confidence of the completion with the current entity or relation, and the entity or relation that receives the highest score is deemed the most probable candidate.

3.2. Poincaré ball model

Hyperbolic geometry is an axiomatic system of geometry with constant negative curvature that is independent of Euclidean geometry. It involves employing exponential maps to transform points from Euclidean space into hyperbolic space, as well as using logarithmic maps to project points from hyperbolic space onto the associated tangent space. The two mappings are shown in Eqs. (1) and (2) as follows:

$$\exp_{\mathbf{0}}^{c_r}(\mathbf{v}) = \tanh(\sqrt{c_r}\|\mathbf{v}\|)\frac{\mathbf{v}}{\sqrt{c_r}\|\mathbf{v}\|} \tag{1}$$

$$\log_{\mathbf{0}}^{c_r}(\mathbf{y}) = \tanh^{-1}(\sqrt{c_r}\|\mathbf{y}\|)\frac{\mathbf{y}}{\sqrt{c_r}\|\mathbf{y}\|} \tag{2}$$

where c_r represents the curvature of the hyperbolic space. The exponential map projects a tangent vector at the origin of the hyperbolic space onto the manifold. This transformation is defined using hyperbolic trigonometric functions to ensure consistency with the underlying geometry. Specifically, the function scales the vector to respect the curvature, preserving the distance metric of hyperbolic space. Similarly, the logarithmic map performs the inverse operation, projecting a point on the manifold back to the tangent space at the origin. This operation utilizes the inverse hyperbolic tangent function to account for curvature-dependent scaling, ensuring that the mapping respects the geometric constraints.

These mappings are crucial for defining operations such as translations, rotations, and embeddings in hyperbolic space, as they enable efficient manipulation of points while maintaining the geometric properties of the space.

In Euclidean geometric space, the translation operation on vectors is performed using vector addition. In this paper, translation in hyperbolic geometric space is achieved using Möbius addition, as shown in Eq. (3):

$$\mathbf{x} \oplus^{c_r} \mathbf{y} = \frac{(1 + 2c_r\mathbf{x}^T\mathbf{y} + c_r\|\mathbf{y}\|^2)\mathbf{x} + (1 - c_r\|\mathbf{x}\|^2)y}{1 + 2c_r\, \mathbf{x}^T\mathbf{y} + c_r^2\|\mathbf{x}\|^2|\mathbf{y}\ \|^2} \tag{3}$$

This study utilizes the Poincaré ball model to depict hyperbolic space, given its suitability for optimization based on gradient methods, Riemannian optimization, etc. [22]. Additionally, it provides benefits like easy parallelization and strong scalability, facilitating the development of efficient embedding algorithms. In hyperbolic space, the distance between points is defined by the length of the geodesic, which represents the distance along the Poincaré line, as shown in Fig. 2, its calculation formula is shown in Eq. (4):

$$d^{c_r}(\mathbf{x}, \mathbf{y}) = \frac{2}{\sqrt{c_r}}\operatorname{arctanh}(\sqrt{c_r}\| - \mathbf{x} \oplus^{c_r} \mathbf{y}\|) \tag{4}$$

4. Proposed methodology

The goal of this paper is to develop a hyperbolic embedding model that performs well in low dimensions. This model should be capable of embedding complex semantic information, such as the symmetry and asymmetry commonly found in knowledge graph data, while preserving the underlying hierarchical structure. The model, MrpHKGE, (1) learns vector representations of knowledge graph data in hyperbolic space to maintain hierarchical structure (Section 4.1), (2) applies suitable rotational transformations in hyperbolic space from both 2D and 3D perspectives to encode logical patterns (Section 4.2), (3) combines these various geometric transformations with an attention mechanism (Section 4.3), and (4) introduces the concept of residuals to the model for translation transformations in hyperbolic space. This complete model is described in Section 4.4.

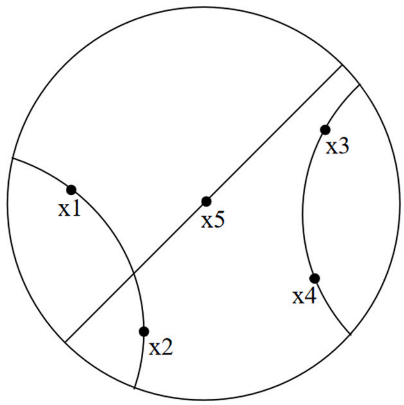

Fig. 2. Poincaré disk geodesics.

4.1. Embedding in hyperbolic space

Research indicates that, compared to Euclidean space, hyperbolic geometry can more effectively represent nonlinear hierarchical structures in low-dimensional spaces [22]. Hyperbolic geometry is characterized by a constant negative curvature, which indicates the extent of deviation from flatness. When the curvature is zero, it aligns with Euclidean geometry. In hyperbolic space, the concept equivalent to a straight line in Euclidean space is known as a geodesic, which is represented as a curve. Theoretically, any finite tree can be nearly perfectly embedded in 2D hyperbolic space [25], a feat that Euclidean space cannot achieve.

Unlike embedding models in Euclidean space where the curvature is fixed at zero, embedding models in hyperbolic space yield different results depending on the curvature of the hyperbolic space being embedded. In contrast to the MuRP model, which embeds all relations into hyperbolic space with the same curvature, our model learns a unique absolute curvature C_r for each different relation, enabling the model to represent different hierarchical structures [23]. The curvature of hyperbolic space determines whether a relation is embedded into a more curved, less flat geometric space or into a flatter, more planar geometric space.

4.2. Hyperbolic transformations

Translation Transformation Translation is a fundamental operation in geometric transformations and plays a significant role in many embedding models based on Euclidean space. In practice, this operation is implemented through vector addition. However, because vector addition lacks a clear definition in hyperbolic space, our model approximates translation transformations in hyperbolic space using Möbius addition [26], as shown in Eq. (3).

2D Rotation 2D rotation can simultaneously model relational patterns such as inversion, composition, symmetry, or antisymmetry, and has been successfully applied in the RotatE model [13]. It plays a crucial role in capturing the multiplicity of data. For example, symmetric relations (e.g., "sibling of") are effectively modeled by rotation matrices that map an entity to itself after a specific angular transformation, while antisymmetric relations (e.g., "parent of") are captured by distinct transformations that maintain directionality. Additionally, inversion and composition patterns are modeled by combining rotations with other transformations to represent multi-hop paths in the graph. In this paper, we use Givens transformations to simulate 2D rotations of relations in hyperbolic spaces with different curvatures [23]. Givens transformations are commonly used 2 × 2 rotation matrices in numerical linear algebra. Assuming an even number of dimensions, we parameterize the rotation using a block-diagonal rotation matrix as shown in Eq. (5).

$$R(\Theta_r) = \mathrm{diag}(G(\theta_{r,1}), G(\theta_{r,2}), \cdots, G(\theta_{r,\frac{n}{2}})), \quad \text{where} \quad G(\theta) := \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix} \tag{5}$$

where $\Theta_r := (\theta_{r,i})_{i \in \{1,\dots \frac{n}{2}\}}$ are parameters specific to each type of relation.

3D Rotation The non-commutative nature of 3D rotation allows the graph embedding model to perform non-commutative compositions. The QuatE model has successfully demonstrated that 3D rotation can be more expressive than 2D rotation for certain datasets [14]. Therefore, this paper applies 3D rotation to the transformation of the head entity in hyperbolic space. This operation is simulated by the Hamilton product matrix operation in 3D space, as shown in Eq. (6):

$$\begin{aligned} v_1 \otimes v_2 &= (x_1x_2 - y_1y_2 - z_1z_2 - t_1t_2) \\ &+ (x_1y_2 + y_1x_2 + z_1t_2 - t_1z_2)\mathbf{i} \\ &+ (x_1z_2 - y_1t_2 + z_1x_2 + t_1y_2)\mathbf{j} \\ &+ (x_1t_2 + y_1z_2 - z_1y_2 + t_1x_2)\mathbf{k} \end{aligned} \tag{6}$$

where the quaternion v consists of one real part and three imaginary units, expressed as: $v = x + y\mathbf{i} + z\mathbf{j} + t\mathbf{k}$, where x, y, z, t being real numbers and $\mathbf{i}, \mathbf{j}, \mathbf{k}$ representing the imaginary units.

4.3. Hyperbolic attention

In the two types of rotational transformations presented in this paper, since they will be applied in hyperbolic spaces with different curvatures, one rotational transformation might perform better under certain conditions. To leverage the advantages of both types of rotational transformations simultaneously, this paper employs an attention mechanism to learn the importance of each rotational transformation.

Let q_1^H and q_2^H be two vectors embedded in hyperbolic space through different rotational transformations. Using the logarithmic map, q_1^H and q_2^H are mapped to the tangent space. The attention scores for the two types of rotational transformations are calculated using Eq. (7):

$$(\alpha_{q_1}, \alpha_{q_2}) = \mathrm{Softmax}(\mathbf{v}^T \log_{\mathbf{0}}^{c_r}(\mathrm{q}_1^{\mathrm{H}}), \mathbf{v}^T \log_{\mathbf{0}}^{c_r}(\mathrm{q}_2^{\mathrm{H}})) \tag{7}$$

where $\mathbf{v}$ is the attention vector. The weighted average is calculated as shown in Eq. (8):

$$Att(q_1^H, q_2^H, \mathbf{v}) := \alpha_{q_1} \log_{\mathbf{0}}^{c_r}(\mathrm{q}_1^{\mathrm{H}}) + \alpha_{q_2} \log_{\mathbf{0}}^{c_r}(\mathrm{q}_2^{\mathrm{H}}) \tag{8}$$

4.4. Scoring function

Building upon the previously obtained modules, incorporating skip connections additionally boosts the embedding model's representational capabilities, resulting in a scoring function based on hyperbolic distance. Specifically, for a triple $(h, r, t) \subseteq \mathcal{V} \times \mathcal{R} \times \mathcal{V}$, let $\mathbf{e}_h^H$, $\mathbf{r}^H$, and $\mathbf{e}_t^H$ be the hyperbolic embeddings of $\mathbf{e}_h$, $\mathbf{r}$, and $\mathbf{e}_t$, respectively. The MrpHKGE model initially performs a fundamental translation transformation on the head entity within hyperbolic space, utilizing Eq. (3), and then applies the transformations from Eqs. (5) and (6) to the head entity's embedding, as shown in Eq. (9):

$$\mathbf{b}_{2D}^H = R(\Theta_r)(\mathbf{e}_h^H \oplus^{c_r} \mathbf{r}^H), \ \mathbf{b}_{3D}^H = (\mathbf{e}_h^H \oplus^{c_r} \mathbf{r}^H) \otimes \mathbf{r}^H \tag{9}$$

Then, using the logarithmic map as shown in Eq. (10), the vectors are mapped back to the tangent space. Using Eq. (8), different embedding results are combined and the concept of residuals is introduced by applying the initial relation matrix r^H to perform a translation transformation in hyperbolic space. Finally, the hyperbolic distance between the resulting embedding and the target tail entity embedding

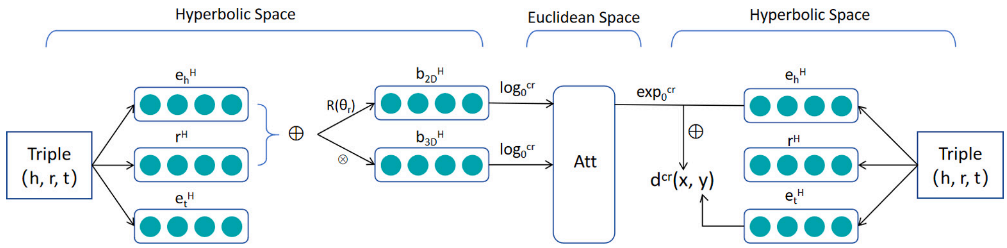

Fig. 3. The structure of the scoring function for the entire model.

is calculated using Eq. (4) to obtain the final scoring function, as shown in Eq. (11):

$$\mathbf{b}_{2D}^{E} = \log_{\mathbf{0}}^{c_r}(\mathbf{b}_{2D}^{H}),\ \ \mathbf{b}_{3D}^{E} = \log_{\mathbf{0}}^{c_r}(\mathbf{b}_{3D}^{H}) \tag{10}$$

$$s(h, r, t) = -d^{c_r}(\exp_0^{c_r}(Att(\mathbf{b}_{2D}^{E}, \mathbf{b}_{3D}^{E}, \mathbf{v}_r) \oplus^{c_r} \mathbf{r}^{H}), \mathbf{e}_t^{H})^2 + b_h + b_t \tag{11}$$

where d_h and d_t are the entity biases acting as boundaries for the scoring function.

The flow of the scoring function for the entire model is illustrated in Fig. 3. Intuitively, the combination of translation and different rotational transformations encodes logical patterns, while hyperbolic space provides sufficient capacity to capture tree-like structures even with limited dimensions. Finally, the similarity between the resulting embedding and the embedding of the target tail entity is compared using hyperbolic distance, yielding the scoring function as shown in Eq. (11).

5. Experiment

Our embedding model achieves good performance even in low-dimensional spaces. In this section, we evaluate the effectiveness of our embedding model by conducting link prediction tasks on knowledge graphs.

5.1. Experimental setup

Datasets We evaluate our proposed model on two public datasets: (1) WN18RR [16] is a subset of the WordNet dataset, containing 40,943 entities and 11 types of relations, with data that naturally has a hierarchical structure. (2) FB15K-237 [27] is a subset of FB15K, which in turn is a subset of the large-scale general-domain knowledge graph Freebase. FB15K-237 includes a total of 14,541 entities and features 237 different types of relations, with data that partially exhibits hierarchy and partially non-hierarchy.

Baselines We compare the MrpHKGE model with state-of-the-art embedding models:

TransE: A well-known translation-based model, TransE represents relations as translations of entities in a low-dimensional vector space. This method works well for modeling simple relationships but struggles with more complex structures, such as hierarchical relations.

RotatE: This model uses rotation-based geometric transformations in the complex plane to model the relationships between entities. It has demonstrated strong performance, particularly for handling symmetric and asymmetric relations.

QuatE: Extending RotatE, QuatE employs quaternion embeddings to model both symmetric and antisymmetric relations in a more flexible way. This allows QuatE to capture richer relational patterns than earlier methods like TransE and RotatE.

MuRP: A more recent method that combines random walk-based strategies with geometric transformations. MuRP has shown strong results in modeling complex, multi-relation patterns within knowledge graphs.

AttH: Like our proposed model, AttH focuses on combining multiple geometric transformations to preserve semantic information to the greatest extent. We select AttH as a baseline due to its shared approach in handling geometric transformations in embedding models.

Evaluation metrics During the experiments, we use the scoring function in Eq. (11) to calculate the similarity between predicted and actual entities. We adopt the following rank-based metrics: (1) MRR assesses the average of the reciprocal ranks assigned to the correct entities, with a higher MRR signifying improved model performance. (2) Hits@n, (where $n \in \{1, 3, 10\}$)evaluates the ratio of correct triples within the top n predicted triples, where higher Hits@n values indicate superior model performance.

Implementation details The configurable hyperparameters of the model include the dimension of the embedding space, optimizer type, negative sampling size, batch size, and learning rate. We select these hyperparameters through grid search and use the test set to choose the best combination of hyperparameters. The main optimizer types we set for the model are Adam and Adagrad.

Although the MuRP model proposed using RSGD for parameter optimization in hyperbolic space, the use of RSGD is complex. Research by Chami et al. [23] shows that applying traditional Euclidean geometric optimization methods in the tangent space of hyperbolic space is equally effective. Therefore, the parameters of our model are first defined in Euclidean geometric space and then mapped to hyperbolic space through exponential transformation for subsequent geometric transformations. Finally, we use traditional Euclidean geometric optimization methods, such as Adam or Adagrad, for parameter optimization.

5.2. Results in low dimensions

This study hypothesizes that the model would perform well in low dimensions, so the training dimension is initially set to 32. The results of the experiments are presented in Table 1. Table 1 compares the performance of the MrpHKGE model with other baseline models, including embedding models in Euclidean space and hyperbolic space. It can be observed that when the training space is set to a low dimension, models based on hyperbolic space embeddings generally outperform those based in Euclidean space.

David Krackhardt [29] and Chami [23] provide two metrics, Khs and ξ_G, to indicate the hierarchical nature of relationships. Generally, the higher the Khs value and the lower the ξ_G value, the more hierarchical the relationship is. To demonstrate the improvement brought by embedding in hyperbolic space for the MrpHKGE model, we conducted a comparative experiment. This experiment compared the Hits@10 metric on the WN18RR dataset for our model when embeddings were performed in hyperbolic space versus Euclidean space. The experimental results, shown in Table 2, indicate that MrpHKGE shows significant improvement in hierarchical relationships such as "hypernym" and "has part", while there is little to no improvement in relationships with minimal hierarchical characteristics such as "similar to" and "verb group".

In the WN18RR and FB15K-237 datasets, the performance of our model matches that of the leading embedding models currently available in hyperbolic space. Moreover, it surpasses several of these advanced hyperbolic embedding models in the H@10 metric on the

Table 1
The results of the link prediction experiments in the low-dimensional space with 32 dimensions are shown below. The highest scores are indicated in bold, and the second-highest scores are underlined. The results for the TransE, RotatE, QuatE, and MuRP models are taken from Ref. [28].

	WN18RR				FB15k-237			
	MRR	H@1	H@3	H@10	MRR	H@1	H@3	H@10
TransE	0.244	0.099	0.350	0.506	0.277	0.194	0.303	0.444
RotatE	0.387	0.330	0.417	0.491	0.290	0.208	0.316	0.458
QuatE	0.445	0.407	0.463	0.515	0.266	0.186	0.290	0.426
MuRP	0.269	0.106	0.402	0.532	0.279	0.196	0.306	0.445
RefH	0.446	0.408	0.462	0.517	0.310	0.224	0.339	0.486
RotH	**0.472**	**0.431**	**0.491**	0.546	0.315	0.224	0.344	0.496
AttH	0.461	0.422	0.477	0.530	**0.322**	**0.234**	0.352	**0.501**
3H-TH	0.468	0.425	0.485	0.548	0.318	0.226	0.349	0.499
MrpHKGE	**0.472**	0.429	**0.491**	**0.551**	0.321	0.233	**0.353**	**0.501**

Table 2
Comparison of Hits@10 for the relations in WN18RR. Higher Khs and lower ξ_G values indicate stronger hierarchical characteristics of the relationship. The MrpKGE model is the Euclidean embedding version of the MrpHKGE model, used to compare the impact of embedding in hyperbolic space on the model's performance. The best score is highlighted in bold.

Relation	Khs	ξ_G	MrpKGE	MrpHKGE	Improvement
member meronym	1.00	−2.90	.398	**.413**	3.77%
hypernym	1.00	−2.46	.235	**.269**	14.5%
has part	1.00	−1.43	.297	**.316**	6.40%
instance hypernym	1.00	−0.82	.510	**.523**	2.55%
member of domain region	1.00	−0.78	**.413**	.395	−4.36%
member of domain usage	1.00	−0.74	**.458**	.448	−2.18%
synset domain topic of	0.99	−0.69	.430	**.437**	1.63%
also see	0.36	−2.09	.666	**.701**	5.26%
derivationally related form	0.07	−3.84	.966	**.971**	0.51%
similar to	0.07	−1.00	**1.00**	**1.00**	0.00%
verb group	0.07	−0.05	**.969**	**.972**	0.31%

Table 3
The results of the link prediction experiments in the high-dimensional space with 500 dimensions are shown below. The best scores are indicated in bold, and the second-best scores are underlined. The results for the TransE, RotatE, QuatE, and MuRP models are taken from [28].

	WN18RR			
	MRR	H@1	H@3	H@10
TransE	0.263	0.107	0.380	0.532
RotatE	0.396	0.384	0.399	0.419
QuatE	0.487	0.442	0.503	0.573
MuRP	0.265	0.105	0.392	0.531
RefH	0.462	0.403	0.481	0.564
RotH	**0.495**	**0.446**	**0.510**	**0.583**
AttH	0.484	0.440	0.497	0.571
3H-TH	0.489	0.444	0.507	0.579
MrpHKGE	0.491	0.441	**0.510**	**0.583**

WN18RR dataset and the H@3 metric on the FB15k-237 dataset. Previous studies have demonstrated that these two public datasets contain rich hierarchical structures [23]. Combined with the experimental results presented in this study, it can be concluded that choosing hyperbolic space for embedding yields significant results in low dimensions.

5.3. Results in high dimensions

The results of our model in the link prediction experiments conducted in high-dimensional space (d = 500) on the WN18RR dataset are shown in Table 3. It is evident that the MrpHKGE model continues to deliver results that are comparable to those of other leading hyperbolic embedding models. However, compared to Euclidean embedding models, hyperbolic embedding models, including MrpHKGE, no longer have a significant advantage. This indicates that when the embedding dimension is sufficiently large, the performance of Euclidean space-based embedding models and hyperbolic space-based embedding models is similar.

In order to more accurately demonstrate the impact of embedding dimensions on the performance of the MrpHKGE model, this section designs an experiment with multiple embedding dimensions. This experiment effectively examines the model's performance under different dimensions and analyzes its efficiency and accuracy in lower-dimensional settings. The embedding dimensions selected for the experiment are $d \in \{10, 16, 20, 32, 50, 200, 500\}$, with most dimensions being low-dimensional and only a few high-dimensional, ensuring a comprehensive evaluation of the adaptability of the MrpHKGE model across various embedding dimensions.

The experiment uses the link prediction task for evaluation. The dataset chosen is WN18RR, a commonly used and representative knowledge graph dataset, capable of effectively testing the model's performance under different embedding dimensions. To measure the model's performance, the metric Mean Reciprocal Rank (MRR) is employed, as it comprehensively reflects the model's accuracy in prediction tasks.

The experimental results, shown in Fig. 4, illustrate the effect of embedding dimensions on the model's performance. The results indicate that with an increase in embedding dimensions, the model performance improves significantly in low-dimensional settings but plateaus at higher dimensions (e.g., 200 and 500). This validates the model's efficiency and accuracy in low dimensions and its performance limits in high dimensions.

6. Conclusions

This paper introduces an innovative embedding model designed to map knowledge graph entities and relationships into hyperbolic space for link prediction tasks. The model leverages the expressive power of hyperbolic space, geometry-based transformations with attention mechanisms, and skip connections to learn low-dimensional KG representations. By incorporating the curvature of hyperbolic space as a trainable parameter, the model can learn unique geometric representations for different relationships and generalize across multiple embedding dimensions. The experimental results demonstrate that the model performs as expected, outperforming traditional Euclidean methods in hierarchical data representation. However, the model does have

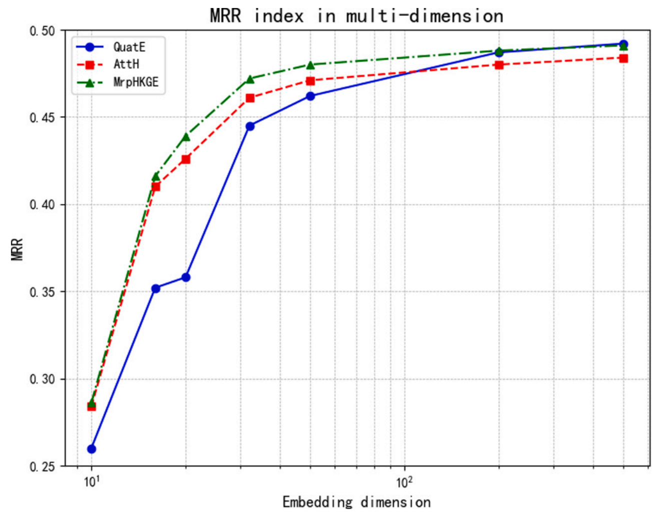

Fig. 4. The performance of each model in multiple dimensions.

some limitations. Its scalability to larger knowledge graphs and its performance in noisy data environments require further investigation. In future work, exploring these aspects will be crucial for enhancing the model's robustness and applicability. Additionally, integrating MrpHKGE with other AI tasks, such as question answering or entity linking, could extend its functionality and broaden its range of applications. In terms of the link prediction method, the strengths of MrpHKGE lie in its ability to efficiently capture hierarchical relationships in low-dimensional spaces and its flexibility across different embedding dimensions. However, the model may face challenges in very large-scale graphs and noisy data, where embedding quality could be impacted. Despite these limitations, the model's ability to adapt to multiple relational patterns and its potential for integration with other AI tasks make it a promising approach for knowledge graph completion and intelligent network applications.

CRediT authorship contribution statement

Longxin Lin: Writing – original draft, Methodology, Investigation. **Huaibin Qin:** Supervision. **Quan Qi:** Supervision. **Rui Gu:** Supervision. **Pengxiang Zuo:** Supervision. **Yongqiang Cheng:** Supervision.

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Declaration of competing interest

The authors declare no conflicts of interest to report regarding the present study.

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