University of Sunderland

Who's manipulating what? Understanding teachers' decisions when choosing manipulatives

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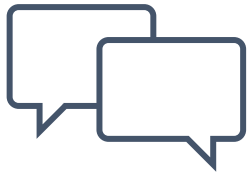
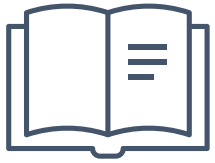
Sixth year of part-time PhD:

How do primary mathematics teachers' perceptions of Teaching for Mastery (TfM) inform their choices when selecting and using manipulatives (concrete resources) within their lessons?

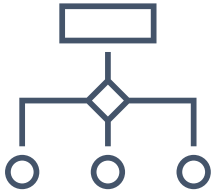
130 questionnaires



14 interviews



descriptive statistics



thematic analysis

13 questionnaire respondents (10%) said they do not use manipulatives

Not encouraged to do so by curriculum lead

I teach Y6, so I generally use visual to support learning

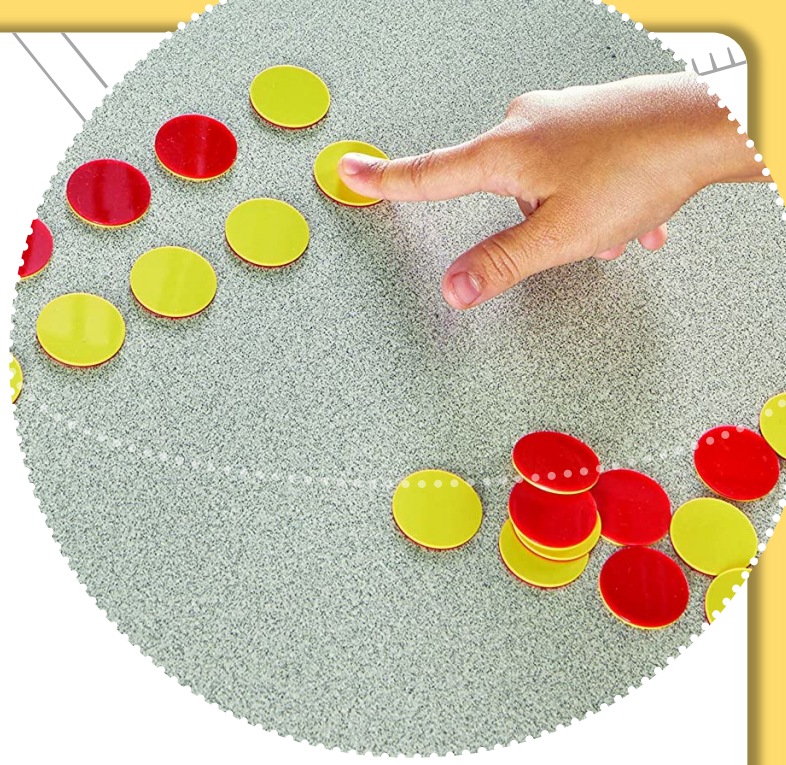
I do use some but usually for demonstration purposes

There is an issue of resources

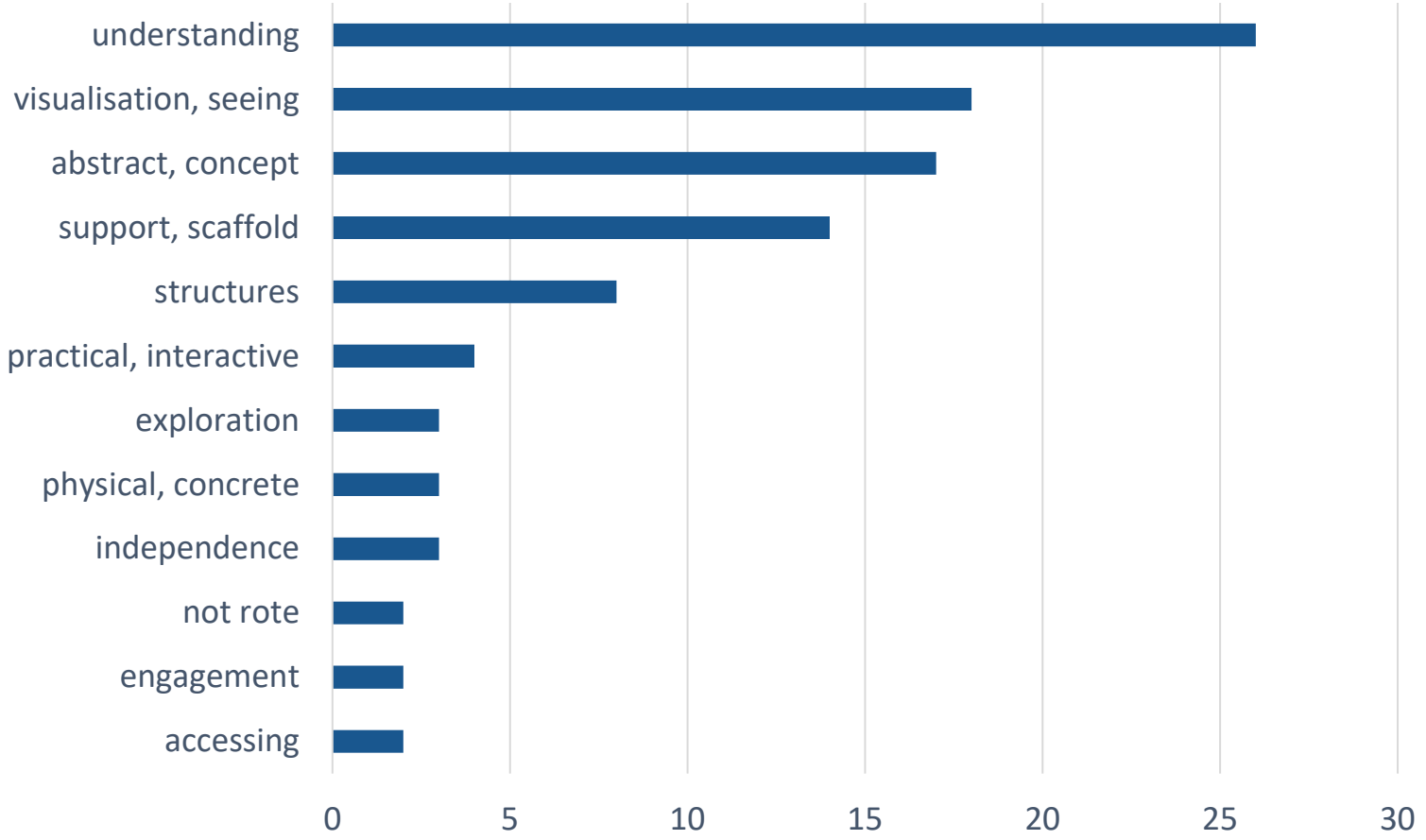
I do demonstrate with manipulations but I do not give them out to the pupils. The pupils tend to get distracted

Which manipulatives are used and how often?

Manipulative	Mean
interlocking cubes, e.g. multilink	4.14
two-sided counters	4.11
Dienes/base ten	3.76
Numicon	3.67
place value counters	3.66
Rekenrek/abacus	3.15
bead strings	3.04
sticks, e.g. lolly sticks	3.01
Cuisenaire rods	2.68
fraction tiles	2.32
algebra tiles	1.94



I use manipulatives in my maths lessons because...



single references:

problem solving; assessment; modelling; progress; inclusion; communication; connections; knowledge; demonstrate

TEN THEMES FROM ANALYSIS

Accessibility and inclusion	• the influence of pupil age, achievement and additional needs
Choice and preference	• factors influencing manipulative decisions
Cognitive processes and understanding	• concepts underpinning the teaching of maths mastery
Concrete-Pictorial-Abstract (CPA)	• the role manipulatives play in this heuristic
Continuing Professional Development (CPD)	• sources, impact on teacher and further needs
Developing the whole child	• the wider benefits of mastery and manipulatives
Notions of TfM and 5 Big Ideas	• links to mastery as defined by NCETM and Maths Hubs
Pedagogy	• teaching principles and pedagogy not exclusive to Maths
Potential limiting factors	• external and internal restrictions including stigma
Schemes	• how commercial and non-commercial schemes influence decisions

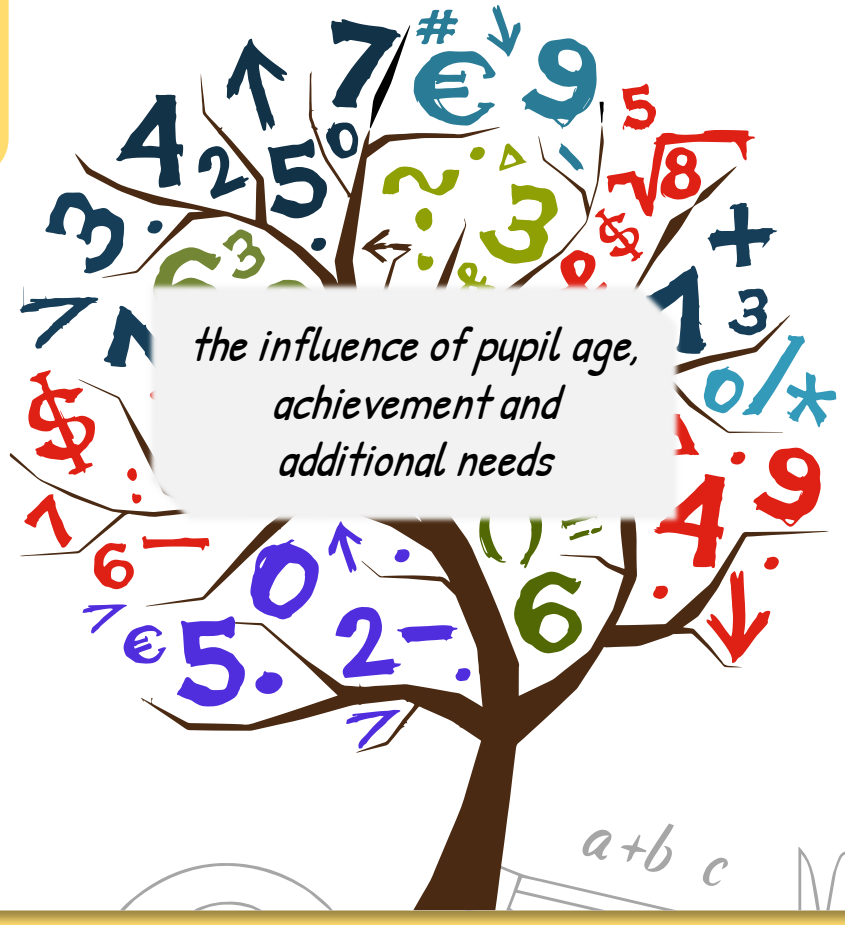
THEME 1: ACCESSIBILITY & INCLUSION

small steps

Mastery for me is where all children are given the opportunity to learn the smaller steps and how they build up (IR9)

'hook' for new concepts

I think it draws them into a lesson very, very well, so I immediately have them hooked if I have things out on the table (IR8)



the influence of pupil age, achievement and additional needs

achieve together

NCETM (2016) 'Essence of Mastery' - all pupils work together on the same lesson content

beneficial for pupil groups

- SEND
- EAL
- EYFS
- Lower achievers

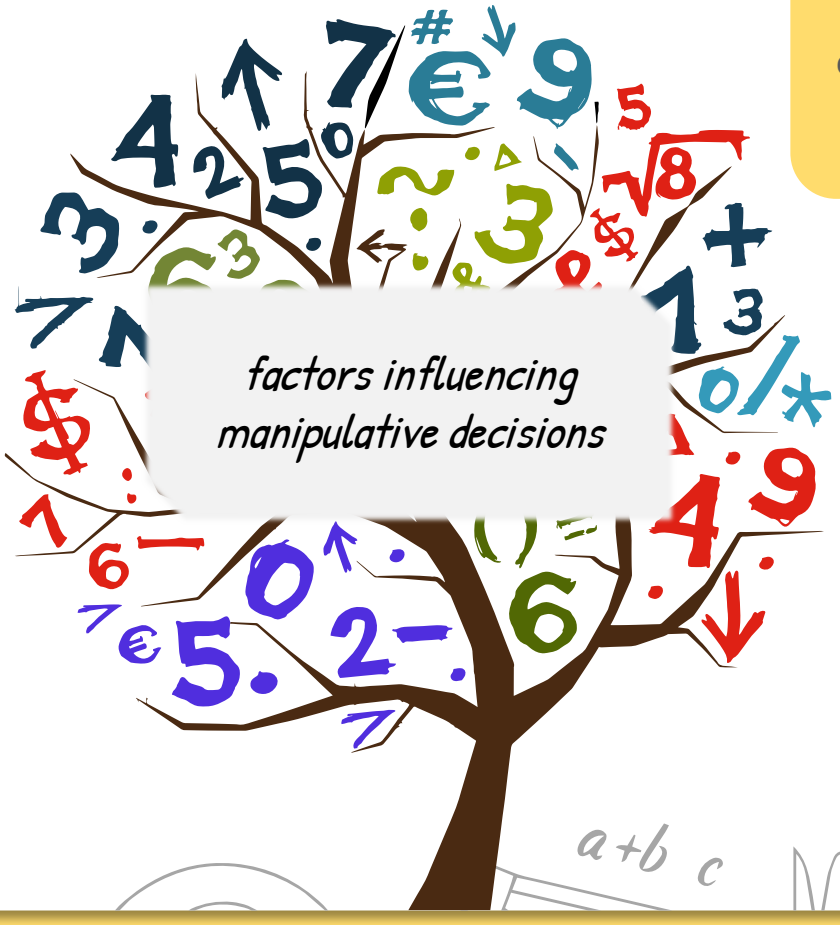
THEME 2: CHOICE & PREFERENCE

curriculum objectives

'we have a tendency to adopt different representations and artefacts depending on their suitability for a specific bit of mathematics' (Back, 2013, p.1)

ease of use

[Dienes] are just easy. They're there. They can grab them. The children are quite happy to use them. They don't take any preparation, really. They're just great (IR7)



factors influencing manipulative decisions

maths topics

You could argue that we use manipulatives more now to teach calculations than other things like number system work (IR5)

versatility

I like counters. I think you can use them in a variety of ways and I think I like the idea of being able to give things different values (IR14)

THEME 3: COGNITIVE PROCESSES & UNDERSTANDING

making connections

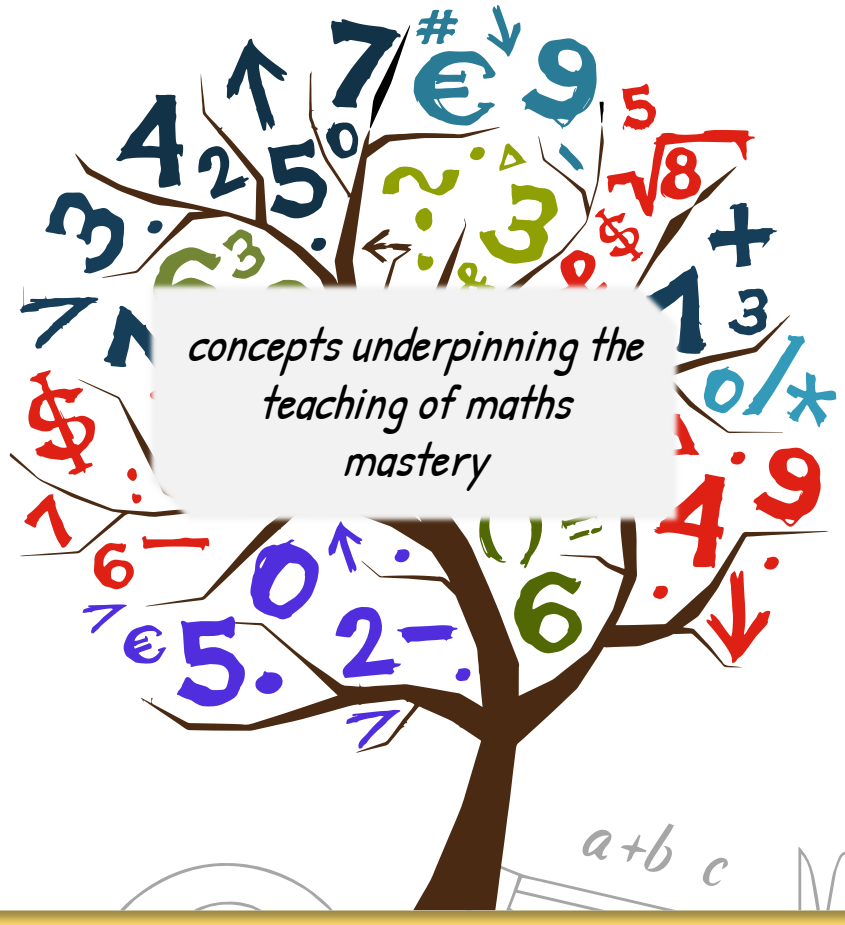
Notions of cognitive constructivism

retention, memory & consolidation

I think [the concept] is easier for them to grasp, but also sort of works with their different type of memory because they're using a different scale of physically doing something (IR13)

prior, new & transferable knowledge

[mastery is] being able to just to master a certain part of maths and use it in different ways, sort of being able to use a certain bit of knowledge elsewhere, knowing when to use it (IR11)



concepts underpinning the teaching of maths mastery

deep understanding

Fostering deep understanding as part of a mastery approach (Drury, 2014)

mental models

[manipulatives] allow students to develop a mental model (QR8)

THEME 4: CONCRETE-PICTORIAL-ABSTRACT (CPA)

perceptual richness

I think [counters] do have a place... but we tend to use more fun things (IR12)

finger counting & sense making

Using both fingers and manipulatives together during arithmetic is an effective way of offloading or externalising the cognitive load (Le et al., 2024)

the role manipulatives play in this heuristic

transition to visualisation

I like it that they can physically use it, and then it's so quick and simple for them to draw (IR6)

notions of representation

Brunerian principles of enactive-iconic-symbolic

abstract thinking

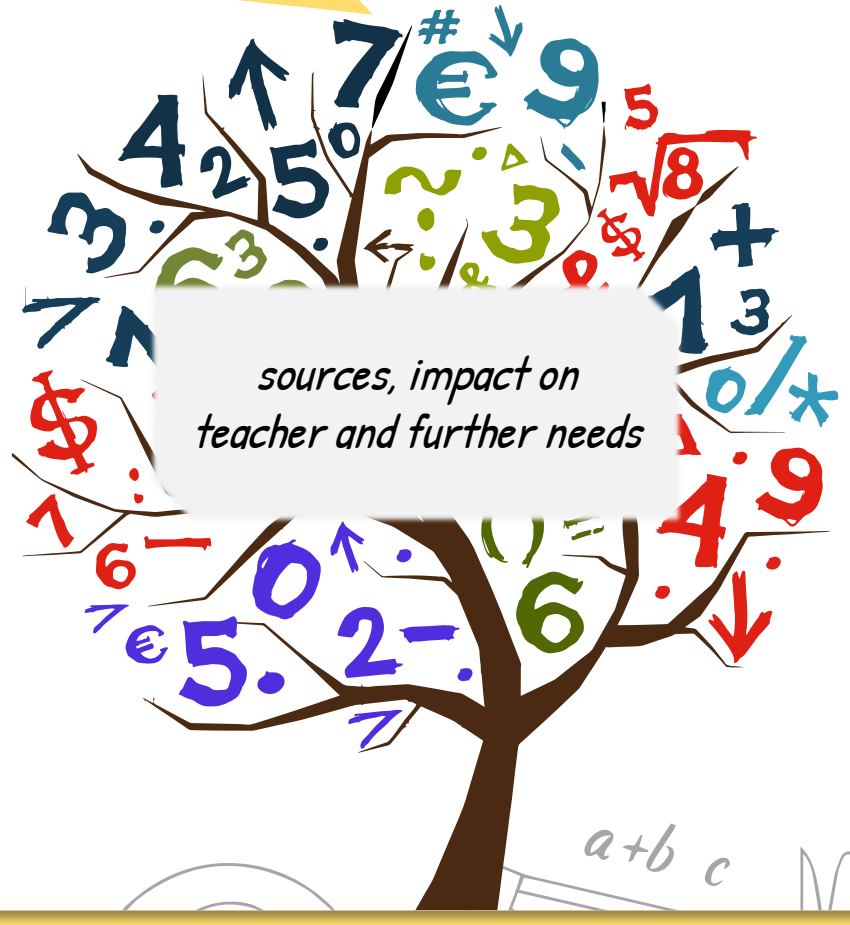
Notions of sense-making (Hoong, Kin & Pien, 2015)

THEME 5: CONTINUING PROFESSIONAL DEVELOPMENT (CPD)

'Teaching for mastery requires a solid base of mathematical pedagogical content knowledge...we have no problem with allowing a great number of teachers with little deep subject knowledge to teach maths to primary-age pupils' (Garry, 2020, pp. 2-17)

facilitating CPD

So there's the monitoring of the school systems, and there's also the coaching side of things through observing other teachers and helping them with their practise (IR5)



sources of CPD

- In-school
- Multi Academy Trust
- Local Authority
- Maths Hub
- Commercial provider

further CPD needs

I'm quite experienced in this year group. If I was to move to another year group... I would definitely need further training I think (IR12)

THEME 6: DEVELOPING THE WHOLE CHILD

exploration & investigation

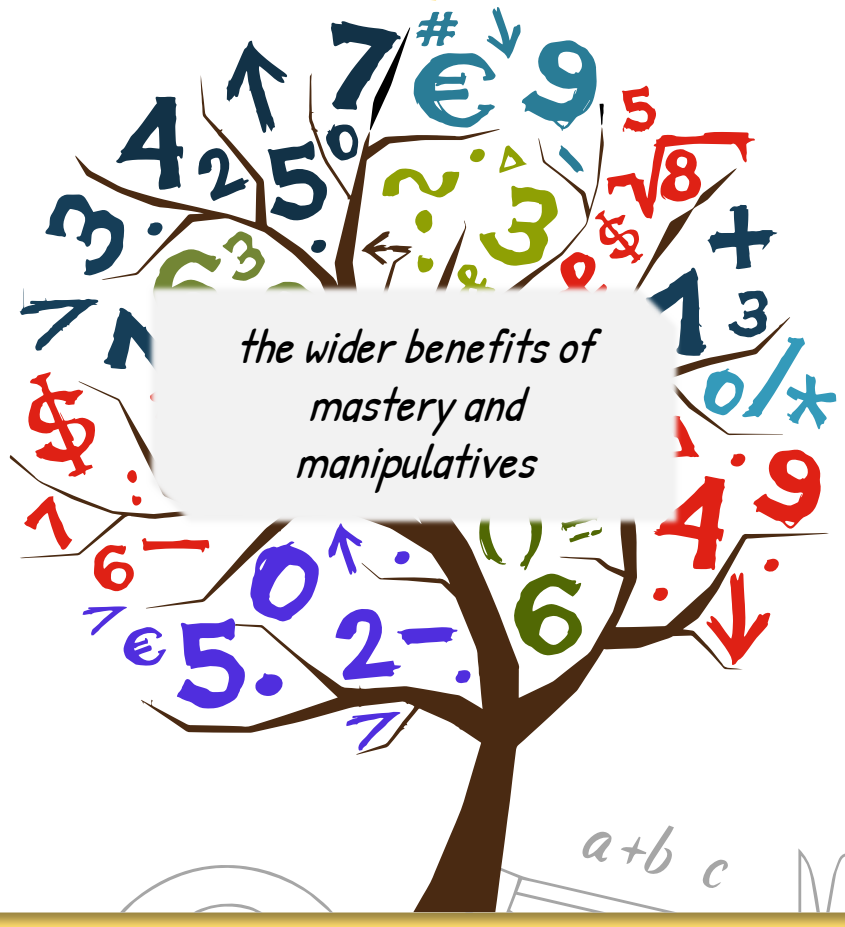
[manipulatives] allow just a bit of exploration, a bit of investigation (IR14)

independence & confidence

independent learning skills
↓
confident problem solving & reasoning

play & fun

I think the children prefer the more fun side to it. You know the colourful (IR12)



the wider benefits of mastery and manipulatives

practical interactivity

children learn by doing and manipulatives are a great way to do that (QR102)

not rote learning

a tonic for didactic approaches?

[manipulatives] deepen their understanding, because they're physically doing it, they're not just listening to me talking (IR12)

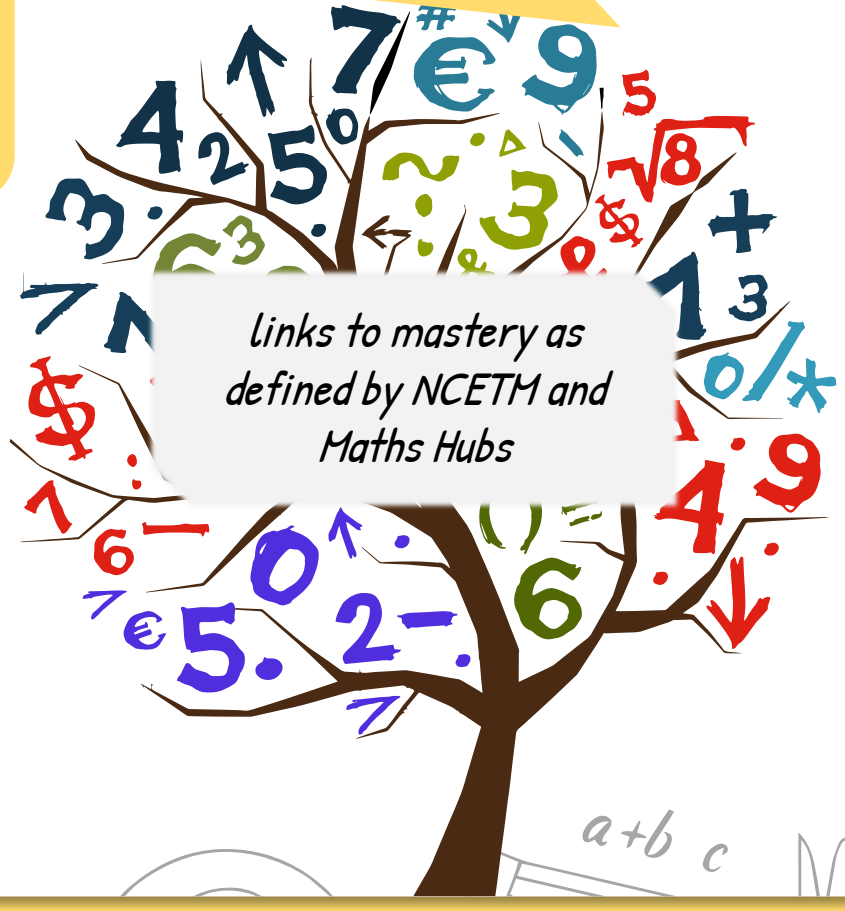
THEME 7: TEACHING FOR MASTERY (TfM) AND 5 BIG IDEAS

fluency, reasoning & problem solving

[manipulatives provide] the pathway for reasoning and explanation that otherwise younger children often find difficult (QR108)

mathematical structures & thinking

[manipulatives] give me a different approach. Rather than just teaching on the board, it gets children to have that mathematical thinking (IR10)



links to mastery as defined by NCETM and Maths Hubs

variation

Conceptual – how a concept is represented
Procedural – how students proceed through a learning sequence

mathematical understanding

Mathematical learning theories imply that using manipulatives can increase conceptual understanding (Griffiths, Back and Gifford, 2017)

dialogue & explanations

e.g. Meaningful Moments for Mathematical Talk (EEF, 2023): Promoting high-quality talk through purposeful, playful practice

THEME 8: PEDAGOGY

assessment

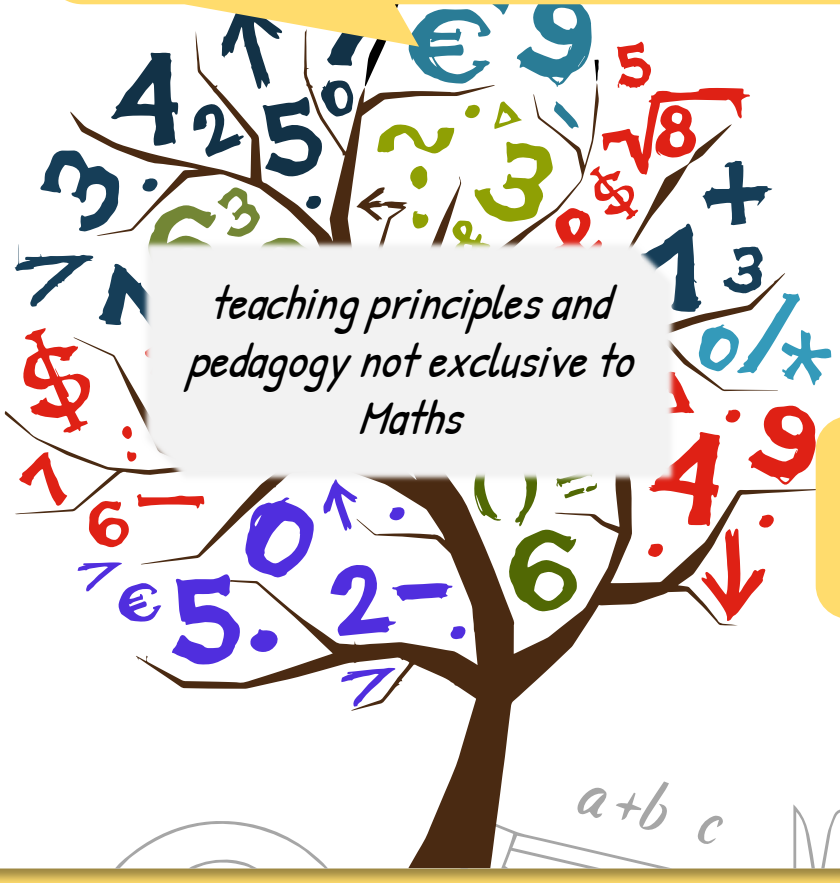
[manipulatives are] really good assessment for learning. I can really clearly see what is going on in their head because it's on the table (IR7)

instruction, guidance & scaffolding

High level of instructional guidance?
Low level?
'Transitioning' level?

stretch & challenge

For whom?
Challenging persistent stigmas



teaching principles and pedagogy not exclusive to Maths

modelling

I use [manipulatives] in lessons at the start of a topic to model the method that they're going to be doing in their head. I think sometimes if you just throw words at the children, they really struggle to internalise what the method is (IR9)

pupil progress

[manipulatives] support, develop and deepen the understanding and progression of children (QR29)

THEME 9: POTENTIAL LIMITING FACTORS

preparation & organisation

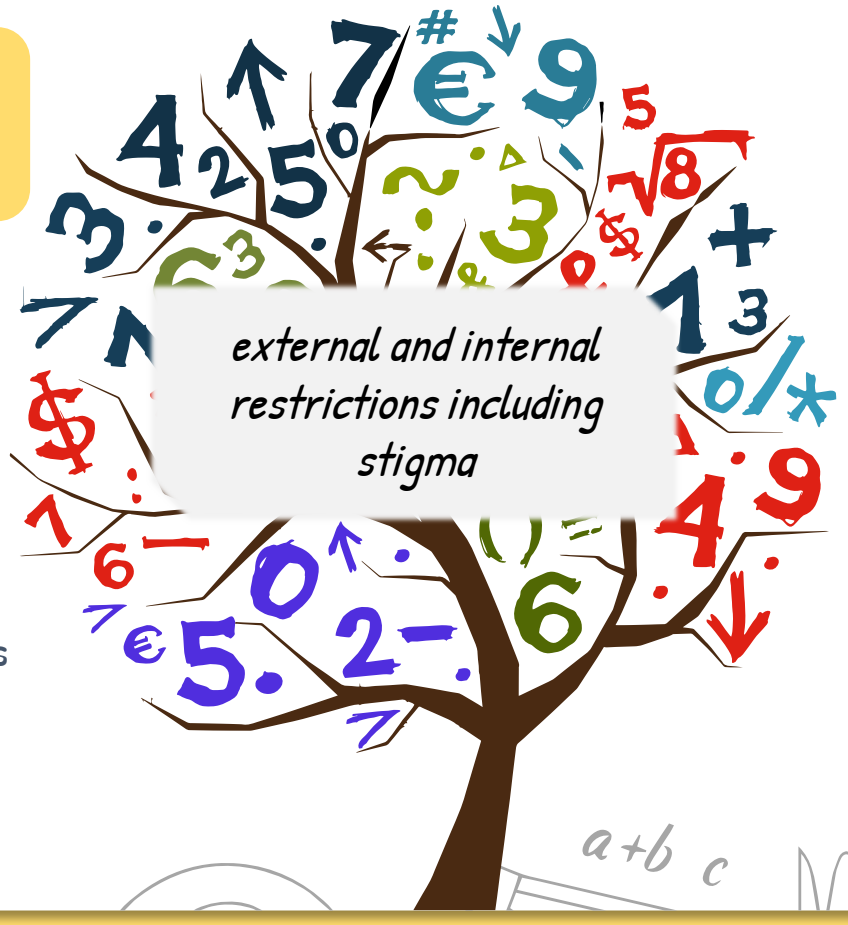
manipulative materials are found in every primary school but often in storerooms rather than classrooms (Swan *et al.*, 2007).

classroom management

I do not give [manipulatives] out to the pupils. They tend to get distracted (QR21)

age of learners

There's the pressure to get through the curriculum obviously in in time for the SATs. You do tend to find you don't have an awful lot of time to have those longer lessons featuring manipulatives (IR5)



external and internal restrictions including stigma

availability & cost

teachers' choice of manipulatives was subject to 'disparate factors' rather than pedagogical principles. The most influential factors were deemed to be 'almost accidental, depending on what was available or had been encountered' (Griffiths, Black and Gifford, 2017, p. 5).

'ability' of learners

there still is this stigma of, 'it's for your lower ability'. And I still think that is generally how people think in school (IR3)

THEME 10: SCHEMES

preparation & organisation

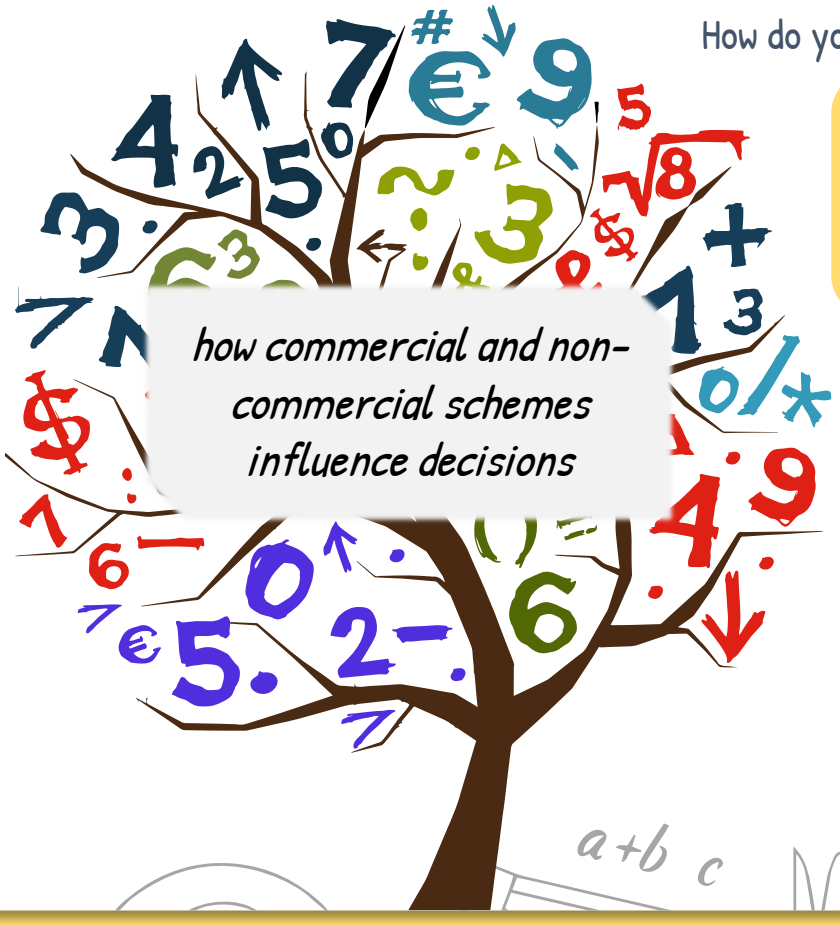
'Good lessons using manipulatives do not just happen. They are the product of much advance thought and preparation' (Stein and Bovalino, 2001, p. 359)

concrete or pictorial?

misconceptions around dual representation: pictorial representations in schemes to replace concrete resources

choice overload

when we originally used White Rose we found it was really broad. So within one lesson it would have five different versions of the same thing and that was just overwhelming for children (IR7)



how commercial and non-commercial schemes influence decisions

ease

How do you decide which manipulative(s) to use in a lesson?

That's quite an easy answer. It's directed by the by the Maths No Problem programme. It's quite precise. You know, you'll need this for this lesson (IR5)

off-the-shelf or adapted

I did encounter times where staff who weren't particularly confident teaching certain areas would deliver White Rose, but not really deal with the misconceptions or struggle to deepen the learning within lessons (IR5)

Practicality *vs.* Pedagogy

AVAILABILITY

What is available in my classroom? Are there sufficient sets for the groups/class? Is it cheap or expensive?

VERSATILITY

Can this manipulative be used for multiple applications? Or is it topic/task specific?

LOGISTICS

Is it ‘easy’ to administer and oversee?

Is it explained in the scheme of work?

CONTENT KNOWLEDGE

Do I understand how to use this manipulative?

PEDAGOGICAL CONTENT KNOWLEDGE

Am I confident instructing others how to use this manipulative for this task?

KNOWLEDGE CREATION

Is the manipulative driving the task?

Is the task driving the manipulative?

Preference?


Thank You
2+2
e=mc²

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